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MATHEMATICS GRADES 10-12

Weighting of Content Areas			
Description	Grade 10	Grade 11	Grade. 12
PAPER 1 (Grades 12:bookwork: maximum 6 marks)			
Algebra and Equations (and inequalities)	30 ± 3	45 ± 3	25 ± 3
Patterns and Sequences	15 ± 3	25 ± 3	25 ± 3
Finance and Growth	10 ± 3		
Finance, growth and decay		15 ± 3	15 ± 3
Functions and Graphs	30 ± 3	45 ± 3	35 ± 3
Differential Calculus			35 ± 3
Probability	15 ± 3	20 ± 3	15 ± 3
TOTAL	100	150	150
PAPER 2: Grades 11 and 12: theorems and/or trigonometric proofs: maximum 12 marks			
Description	Grade 10	Grade 11	Grade 12
Statistics	15 ± 3	20 ± 3	20 ± 3
Analytical Geometry	15 ± 3	30 ± 3	40 ± 3
Trigonometry	40 ± 3	50 ± 3	40 ± 3
Euclidean Geometry and Measurement	30 ± 3	50 ± 3	50 ± 3
TOTAL	100	150	150

Mathematics in the FET

The subject Mathematics in the Further Education and Training Phase forges the link between the Senior Phase and the Higher/Tertiary Education band. All learners passing through this phase acquire a functioning knowledge of the Mathematics that empowers them to make sense of society. It ensures access to an extended study of the mathematical sciences and a variety of career paths.

In the FET Phase, learners should be exposed to mathematical experiences that give them many opportunities to develop their mathematical reasoning and creative skills in preparation for more abstract mathematics in Higher/Tertiary Education institutions.

Extracted from:

*National Curriculum Statement (NCS)
Curriculum and Assessment Policy Statement
Further Education and Training Phase
Grades 10-12*



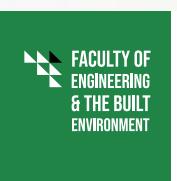
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Compiled by / Saamgestel deur:
Dr Gerrit Stols

Euclidean Geometry Compiled by / Euklidiese Meetkunde Saamgestel deur:
Ria Brown & Philip R. Brown



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Paper 1 / Vraestel 1

- Patterns and sequences / Getalpatrone
- Annuities and finance / Annuitete en finansies
- Functions and graphs / Funksies en grafieke
- Algebra and equations / Algebra en vergelykings
- Calculus / Differensiaalrekenen

Products & Factorisation / Produkte & Faktorisering

Products / Produkte

General cases / Spesiale gevalle

- $a(b + c) = ab + ac$
- $(a + b)(c + d) = ac + ad + bc + bd$
- $(a + b)(d + e + f) = ad + ae + af + bd + be + bf$

Special cases / Spesiale gevalle

- Difference of squares / Verskil van vierkante: $(2a^2 + 3b)(2a^2 - 3b) = 4a^4 - 9b^2$
- Square / Vierkant: $(a + 3b)^2 = a^2 + 6ab + 9b^2$

Factorisation / Faktorisering

Step 1: Find the common factor / Haal gemeenskaplike faktor uit:

Step 2: Count the number of terms / Tel die aantal terme

2 Terms / Terme

- Different of squares: $x^2 - y^2 = (x - y)(x + y)$
- Sum of cubes: $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
- Difference of cubes: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

3 Terms / Terme: $6x^2 - 19x + 10 = (2x - 5)(3x - 2)$

4 Terms / Terme: Grouping / Groepeer 2-2 terms: $x^2 - xp - xq + pq = x(x - p) - q(x - p) = (x - p)(x - q)$

Use factor theorem / Gebruik die faktorstelling



Worksheet / Werkkaart 1

1. Simplify the expressions/Vereenvoudig die uitdrukings:

- a) $-2x - (x - 2x^2)$
- b) $(x - 2x^2) - 2x$
- c) $(x - 2x^2)(-2x)$
- d) $-2x(x - 2x^2)$
- e) $-2x + (x - 2x^2)$
- f) $(-2x)(x - 2x^2)$
- g) $(2x - y)^3$
- h) $(x + 2)(x^2 - 2x + 4)$
- i) $(4x^2 + 6x + 9)(2x - 3)$

2. Factorise/Faktoriseer:

- a) $x^2 + x - 12$
- b) $x^2 - x - 12$
- c) $x^2 + x + 12$
- d) $x^2 - x + 12$
- e) $x^2 - 7x + 12$

3. Simplify the expressions/Vereenvoudig die uitdrukings:

- a) $\frac{2x}{5x^2 - 25x} + \frac{x+2}{25-x^2}$
- b) $\frac{a-b}{a+b} \div \frac{a^2 - 2ab + b^2}{a^3 + b^3}$
- c) $\frac{x}{x^2 - 4} - \frac{2x+1}{x^2 - 2x}$
- d) $\frac{2x^2}{x^2 - 1} - \frac{x^3 - 1}{x - 1}$



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Indices & surds / Eksponente & wortelvorme

Definition: $\sqrt{a} = b \Leftrightarrow b^2 = a$ and $a \geq 0$ & $b \geq 0$

$$\therefore \sqrt{a} = a^{\frac{1}{2}} ; a^{\frac{1}{n}} = \sqrt[n]{a} ; \frac{a}{x^{\text{even number}}} < 0$$

Exponential laws Eksponentwette	Definitions Definisies	Surds Wortelvorme
$a, b \in R$ and $n, m \in No$ $a^k \cdot a^s = a^{k+s}$ $\frac{a^k}{a^s} = a^{k-s}$ $(a^m)^n = a^{mn}$ $a^k \cdot b^k = (a \cdot b)^k$	$a^0 = 1$ $1^n = 1$ $a^{-n} = \frac{1}{a^n}$	$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$



Worksheet / Werkkaart 2

1. Are the following true or false? / Is die volgende waar of vals?

a) $2^4 \cdot 2^5 = 4^{4+5} = 4^9$

b) $2^2 + 2^3 = 2^5$

c) $(-2)^2 = -2^2$

2. Simplify without using a calculator / Vereenvoudig sonder om 'n sakrekenaar te gebruik:

a) $\left(\frac{8}{27}\right)^{-\frac{2}{3}}$

b) $\frac{2^{x-3} - 2^x}{2^{x+1}}$

c) $\sqrt{9^3} + 8^{-\frac{2}{3}}$

d) $\frac{2 \cdot 5^{x+1} + 3 \cdot 5^{x+2}}{4 \cdot 5^{x+1} - 3 \cdot 5^x}$

e) $\frac{8^{n-3} \cdot 32^{-n+1} \cdot 6^{2n}}{9^n}$

f) $\frac{2^{x+1} \cdot 8^{x-1}}{4^{2(x-2)}}$

3. Simplify without using a calculator / Vereenvoudig sonder om 'n sakrekenaar te gebruik:

a) $\frac{\sqrt{8} + \sqrt{8}}{\sqrt{2} \cdot \sqrt{8}}$

b) $\sqrt{32} - \sqrt{288} + \sqrt{800}$

c) $\sqrt{2} \cdot \sqrt{8}$

d) $\frac{\sqrt{18} - \sqrt{32}}{\sqrt{8}}$

e) $\sqrt[3]{4 \cdot \sqrt[3]{16}}$

f) $\frac{\sqrt{27} \cdot \sqrt{18} \cdot \sqrt{32}}{\sqrt{12} \cdot \sqrt{8}}$

g) $\sqrt{3}(\sqrt{48} - 3\sqrt{75} + 2\sqrt{108})$

4. Rationalise the denominator / Rasionaliseer die noemer:

a) $\frac{2}{\sqrt{2}}$

b) $\frac{5}{-2\sqrt{3}}$

Quadratic equations / Kwadратiese vergelykings

Methods to find the roots / Metodes om die wortels te bepaal:

WRITE IN STANDARD FORM & Get rid of fractions: $\times \text{LCM}$ (denominators $\neq 0$)

- Factorising / Faktorisering
 - i. One side = 0 / Een kant = 0
 - ii. Factorise / Faktorisear
 - iii. Let (...) = 0 or (...) = 0 / Stel (...) = 0 of (...) = 0
- Formula / Formule: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Completing the square / Vierkantsvultooiing
 - i. Divide by the coefficient of x^2 / Deel deur koëffisiënt van x^2
 - ii. Take constant term to right-hand side / Konstante term na regterkant
 - iii. Add $\left(\frac{\text{Coef.of } x}{1} \times \frac{1}{2}\right)^2$ to both sides / Tel albei kante by $\left(\frac{\text{Koëf. van } x}{1} \times \frac{1}{2}\right)^2$
 - iv. LHS: Factorise & RHS: LCM / LK: Faktorisear & RK: KGV
 - v. Extract $\pm \sqrt{\quad}$ / Trek $\pm \sqrt{\quad}$
 - vi. Get x alone on a side and LCM / LK: Kry x alleen & RK: KGV



Worksheet / Werkkaart 3

1. Solve for x / Los op vir x :

a) $2x^2 + x - 6 = 0$

b) $(2x - 3)(x - 1) = 15$

c) $\frac{4}{x+3} = \frac{x-3}{4}$

d) $(x+3)(x-5) = 9$

e) $5x^2 - 80 = 0$

f) $x - 18 - 6x(x - 5) = 0$

g) $1 + \frac{2}{x+4} = 3 - \frac{x-2}{x+4}$

h) $\frac{3x-2}{2x-3} + 2 = \frac{5x}{x+4}$

i) $\frac{1+\frac{1}{x}}{1-\frac{1}{x}} = 1$

2. Solve for x by completing the square / Los op vir x d.m.v. vierkantsvoltooiing:

a) $2x^2 + 14x + 24 = 0$

b) $2x^2 + x - 6 = 0$

c) $ax^2 + bx + c = 0$

d) $mx^2 + x^2 - x + m = -1$

3. Solve for x , using the formula / Los op vir x deur die formule te gebruik.

a) $3 - x^2 = 2x^2 + 1$

b) $x - \frac{4}{x} + 2 = 0$

4. Factorise / Faktoriseer:

a) $x^3 - 3x^2 + 4x - 12$

b) $2x^3 + x^2 - 13x + 6$

c) $x^3 + 6x^2 - 9x - 14$

d) $x^3 + 3x^2 - 3x - 9$

5. Solve for x / Los op vir x :

a) $4x^5 - 3x^4 - 4x + 3 = 0$

b) $-x^3 + 3x = 2$

c) $-x^3 + 5x^2 + 8x = 12$

6. Solve for x if/ Los op vir x as: $(2x + 3)(x^2 - 5) = 0$ and/en

a) $x \in \mathbb{Z}$

b) $x \in \mathbb{Q}$

7. Given/Geggee: $\sqrt{-1} = i$

Write the roots of $x^2 + 2x + 5 = 0$ in terms of i .

Skryf die wortels van $x^2 + 2x + 5 = 0$ in terme van i .

Simultaneous equations / Gelykydige vergelykings

Methods / Metodes:

- **Elimination / Eliminasie**

Use this method if there is no term with xy . / Gebruik hierdie metode indien daar geen term met xy is nie.

- i. Write similar terms underneath each other / Skryf gelyksoortige terme onder mekaar neer
- ii. Get coefficients of x or y to be the same by \times / Kry koëffisiënte van x of y dieselfde
- iii. Eliminate one variable / Elimineer een onbekende
- iv. Solve equation / Los op
- v. Substitute back and determine other variable / Vervang terug en bepaal ander onbekende

- **Equate equations / Stel vergelykings gelijk**

Use this method if you can get y alone on a side of both equations.

Gebruik hierdie metode indien y alleen aan 'n kant van albei vergelykings gekry kan word.

- **Substitution / Substitusie**

If you can't use method 1 or 2, this method always works. / Hierdie metode sal altyd werk, indien metode 1 of 2 nie gebruik kan word nie.

- i. Look for simplest equation / Soek eenvoudigste vergelyking
- ii. Get x or y alone (preferably not fraction) / Kry x of y alleen (verkieslik nie breuk nie)
- iii. Substitute in other equation and determine x or y / Vervang in ander vgl. en bepaal x of y .
- iv. Substitute answer into any of the original equations to determine the other variable / Vervang antwoord om ander onbekende te bepaal.



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Inequalities / Ongelykhede

Step 1: One side = 0 / Een kant = 0

Step 2: Factorise / Faktoriseer

Step 3: Calculate roots / Bereken wortels: Let $(\ldots) = 0$ or $(\ldots) = 0$ / Stel $(\ldots) = 0$ of $(\ldots) = 0$

Step 4: Make a rough sketch of parabola / Maak rowwe skets van die parabol

Step 5: If $(***)(***) < 0$ read off the x values of the graph under the x -axis
 As $(***)(***) < 0$ lees dan die waardes van die grafiek af onder die x -as.

If $(***)(***) > 0$ read off the x values of the graph above the x -axis
 As $(***)(***) > 0$ lees dan die waardes van die grafiek af bo die x -as.

Example: Solve for x / Los op vir x : $x^2 - x \geq 6$

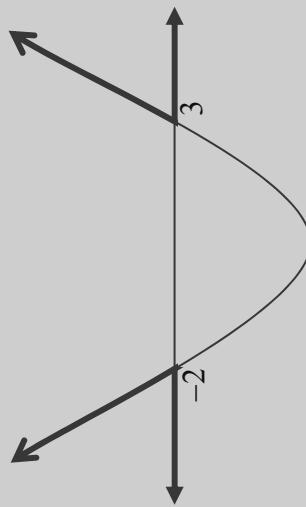
Step 1: $x^2 - x - 6 \geq 0$

Step 2: $(x - 3)(x + 2) \geq 0$

Step 3: x -intercepts / x -afsnitte: 3 and -2

Step 4: See sketch / Sien skets

Step 5: Solution / Oplossing: $x \leq -2$ or / of $x \geq 3$



Worksheet / Werkkaart 4

1. Solve for x / Los op vir x :

a) $2^x = 32$

b) $2^x = 5$

c) $5^x - 100 = 0$

d) $3 \cdot 2^{x+1} - 18 = 0$

e) $3 \cdot 4^x - 8\sqrt{18} = 0$

f) $3^{2x+1} = 12$

g) $x^{\frac{1}{2}} = 4$

h) $5 \cdot 2^{x+1} - 8 = 32$

i) $2 \cdot 2^x - 64 = 0$

2. If $A = P(1+i)^n$, make n the subject of the formula / Maak n die onderwerp van die formule.

3. If $F = \frac{x[(1+i)^n - 1]}{i}$, make n the subject of the formula / Maak n die onderwerp van die formule.

4. Determine the points of intersection of the curves of:

Bepaal die snypunte van die krommes van:

$$y + 2 = 2x \text{ and/en } y = x^2 - 4x + 3$$

5. Solve simultaneously for x and y / Los gelyktydig vir x en y op:

a) $2x + 3y = 2$ and/en $x - 4y = 12$

b) $3y - x - 4 = 0$ and/en $x^2 - xy + y^2 = 3$

c) $y + 2x = 2$ and/en $y^2 + 2x^2 = 3xy$

d) $y^2 + 2x^2 - 3xy = 0$ and/en $y + 2x = 2$

6. Solve for x / Los op vir x :

a) $1 - 2x > 0$

b) $(x + 1)(x - 2) > 0$

c) $(2x - 3)^2 < 4$

d) $x^2 - x - 6 \geq 0$

e) $x^2 < x + 12$

f) $x^2 - 2x \leq 8$

g) $x^2 < 16$

h) $x^2 \geq 9$



Patterns & sequences Pattern / Getalpatrone & rye

Abbreviations / Afkortings

- a first term of the sequence / eerste term van die rye ($a = T_1$)
- d constant difference / konstante verskil (RR): $d = T_2 - T_1$ or $d = T_{30} - T_{29}$
- r constant ratio (GS) / konstante verhouding (MR): $r = \frac{T_2}{T_1}$ or $r = \frac{T_{45}}{T_{44}}$
- n the number of terms / die aantal terme
- T_n the value of the nth term of the sequence / die waarde van die n de term van die rye
- S_n the sum of the first n terms / die som van die eerste n terme

Patterns

- **Exponential:** $1; 2; 4; 8; \dots = 2^0; 2^1; 2^2; 2^3; \dots$ or $3^0; 3^1; 3^2; 3^3; \dots$
- **Arithmetic:** If 1st difference is constant $\Rightarrow T_2 - T_1 = T_3 - T_2$
- **Geometric:** If the ratio is constant $= \frac{T_2}{T_1} = \frac{T_3}{T_2}$
- **Recursive sequences**
 - $1; 1; 2; 3; 5; 8; \dots \Rightarrow$ Fibonacci sequence: $T_{n+1} = T_n + T_{n-1}$
 - $1; 9; 33; 105; 321; \dots \Rightarrow T_n = 3(T_{n-1} + 2)$
- **Quadratic:** If 2nd difference is constant: $T_n = p.n^2 + q.n + r$
- **Combination:** $\frac{3}{1}; \frac{5}{2}; \frac{7}{3}; \frac{9}{4}; \dots \Rightarrow T_n = \frac{2n+1}{n}$

Arithmetic sequences / Rekenkundige rye:

- 2; 5; 8; ... of $-3; -7; -11; \dots$
- 2) TEST / TOETS: $T_2 - T_1 = T_3 - T_2$
- 3) $T_n = a + (n-1)d$
- 4) $S_n = \frac{n}{2}[2a + (n-1)d]$
- 5) $S_n = \frac{n}{2}[a + l]$

Geometric sequences / Meetkundige rye:

- $2; 1; \frac{1}{2}; \dots$ of $2; -4; 8; \dots$
- 1) TEST / TOETS: $\frac{T_2}{T_1} = \frac{T_3}{T_2}$
- 1) $T_n = ar^{n-1}$
- 2) $S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$
- 3) $S_\infty = \frac{a}{1-r}$ as $|r| < 1$

Worksheet / Werkkaart 5

- Determine the unit's digit (i.e. last digit of the number) of the product of the first 200 prime numbers.
 - A rubber ball is bounced from a height of 90 m and after each successive bounce loosed 30% of the previous height.
 - What maximum height will the ball reach during the 5th bounce?
 - After how many rebounces will the ball reach a height of less than 3 metres?
 - Calculate the total distance travelled by the ball after it came to rest.
 - The following number pattern was given 1; 5; 11; 19; ...
 - Determine the 5th term in the pattern.
 - Derive a formula for the n th number in the pattern.
 - What is the 50th number in the pattern?
 - A swimming pool company builds rectangular pools and uses square paving blocks around it.
 - How many square paving blocks are needed for an 8×5 pool?
 - How many square paving blocks are needed for a $m \times n$ pool?
 - How many square paving blocks are needed for a $m \times m$ pool?
 - What is the largest pool that can be built with 200 square paving blocks?
1. Bepaal die ene-syfer van die produk van die eerste 200 priemgetalle.
2. 'n Rubberbal wat van 'n hoogte van 90 m laat val word, verloor met elke terugsprong 30% van sy vorige hoogte.
- Wat is die maksimum hoogte wat die bal tydens die 5de terugsprong bereik?
 - Na hoeveel spronge bereik die bal 'n hoogte van minder as 3 meter?
 - Bereken die totale afstand wat die bal aflê het nadat dit tot rus gekom het.
3. Die volgende getal patroon word gegee: 1; 5; 11; 19; ...
 - Bepaal die 5de term in die patroon.
 - Lei 'n formule af vir die n -de getal in die patroon.
 - Wat is die 50ste getal in die patroon?
4. 'n Swembad maatskappy bou reghoekige swembaddens en gebruik vierkantige blokke om die swembad as plaveisel.
- Hoeveel vierkantige plaveisel blokke word benodig vir 'n 8×5 swembad?
 - Hoeveel vierkantige plaveisel blokke word benodig vir 'n $m \times n$ swembad?
 - Hoeveel vierkantige plaveisel blokke word benodig vir 'n $m \times m$ swembad?
 - Wat is die grootste swembad wat met 200 vierkantige plaveisel blokke gebou kan word?
- | | | | | | | | | |
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Annuities / Annuiteite

Home loans (bond repayment) / Huislenings (Verbandpaaiemende)

Interest on a home loan is calculated at the beginning of each month, and payments are calculated on a monthly basis. / Rente op 'n huislening word bereken aan die begin van elke maand, en betalings word op 'n maandelikse basis bereken.

Use Present value formula to calculate x / Gebruik die Huidige waarde formule om x te bereken: $P = \frac{x[1 - (1 + i)^{-n}]}{i}$

P = Loan (Bond amount) / Lening (Verband bedrag)

n = number payments / aantal terugbetaalings (aantal paaiemende)

i = interest rate per period / rentekoers per saamgestelde periode

x = regular payment / gereelde betaling

Total amount to be repaid / Totale bedrag wat terug betaal moet word:

Regular payment \times Number of repayments / Paaiemende \times Aantal terugbetaalings

Outstanding balance / Uitstaande balans:

Working from the present time to the point in time when the balance must be calculated / Werk vanaf beginnyd tot die tydstip waarop die balans bereken moet word: Balance = (loan + interest) – (sum of payments + interest) Balans = (lening + rente) – (som van paaiemende + rente)

Sinking funds / Sinkende fondse

Definition: Start saving a fixed monthly instalment to provide for the purchase of new equipment in future. Use present equipment as a deposit.

Definisie: Begin vroegtydig 'n gereelde paaiemende spaar om voorsiening te maak vir die aankoop van nuwe toerusting in die toekoms. Verkoop huidige toerusting vir 'n deposito

Monthly instalment: Calculate x in Future Value Formula / Maandelikse paaiemende: Bereken x in die Toekomstige waarde formule $F = \frac{x[(1 + i)^n - 1]}{i}$

Worksheet / Werkkaart 6

1. Tokyo borrowed R50 000 and agrees to repay the loan by means of 50 equal monthly instalments. Interest is calculated at 16% per annum compounded monthly. Calculate the monthly instalment.
2. An amount of x rand is invested k times per annum in an annuity. The interest rate is $r\%$ per annum and is compounded k times per annum. Derive a formula from first principles that you can use to calculate the amount available in the annuity after n years.
3. Tabu decides to invest R5 000 quarterly in an annuity. His first payment is only at the end of the first quarter. If the compound interest he earns is calculated quarterly at 16% per annum, calculate the value of this annuity in five years time from now.
4. The cost of a bus is R2 400 000. The cost is expected to rise at a rate of 16% per annum compound interest, while the value of the bus depreciates at a rate of 22% compounded depreciation per annum. The life expectancy of the bus is 5 years.
 - a) Find the scrap value of the bus.
 - b) Find the cost of a new bus in five years' time.
 - c) A sinking fund is established to pay for the new bus. For the sinking fund, payments are made into an account paying 16% per annum, compounded monthly. Find the value of the sinking fund and the size of the monthly payments, if payments start in a year's time and stop the month before the purchase of the new bus.

1. Tokyo leen R50 000 en kom ooreen om die lening in 50 gelyke maandelikse paaiemente terug te betaal. Rente word bereken teen 16% per jaar maandeliks bereken. Bereken die maandelikse paaiment.
2. 'n Bedrag van x rand word k keer per jaar in 'n annuitet belê. Die rentekoers is $r\%$ per jaar en word k keer per jaar saamgestel. Lei 'n formule vanuit eerste beginsels af waarmee u die bedrag beskikbaar in die annuitet na n jaar sal kan bereken.
3. Tabu besluit om kwartaalliks R5 000 in 'n annuitet te belê. Sy eerste betaling is eers aan die einde van die eerste kwartaal. As die saamgestelde rente wat hy verdien teen 16% per jaar kwartaalliks gekapitaliseer word, bereken die waarde van hierdie annuitet oor vyf jaar van nou af.
4. 'n Bus kos R2 400 000. Die verwagte styging in koste is 16% saamgestelde rente per jaar. Die waarde van die bus verminder teen 'n saamgestelde koers van 22% per jaar. Die verwagte gebruikstyd van die bus is 5 jaar.
 - a) Bepaal die skrootwaarde van die bus.
 - b) Vind die koste van die nuwe bus oor vyf jaar.
 - c) 'n Delgingsfonds word gestig om vir die nuwe bus te betaal. Vir hierdie fonds geld die volgende: Die paaiemente word gemaak in 'n rekening wat 16% per jaar betaal, maandeliks saamgestel. Bepaal die waarde van hierdie delgingsfonds en die grootte van die maandelikse paaiment, indien paaiemente oor een jaar begin en eindig 'n maand voordat die bus aangekoop word.



Straight line & parabola / Reguitlyn & parabool

Standard form / Standaardvorm: $y = mx + c$

- m (slope / gradiënt) = $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$
- || lines / lynne: $m_1 = m_2$
- \perp lines / lynne: $m_1 \cdot m_2 = -1$
- c is the y -intercept / c is die y -afsnit

Sketch (3 methods) / Skets (3 metodes)

- Table: Choose x and determine y / Tabel: Kies x en bepaal y
- Double intercept / Dubbel-afsnit metode:
 - y -intercept: let $x = 0$ / y -afsnit: stel $x = 0$
 - x -intercept: let $y = 0$ / x -afsnit: stel $y = 0$
- Gradient intercept / Gradiënt afsnit:
 - Get y -intercept / Kry y -afsnit
 - Use m : Work in y then in x direction / Gebruik m : Werk eers in y -en dan in x -rigting

Determining the equation / Bepaling van die vergelyking

- Determine gradient of line / Gebruik die gradiënt van die lyn
- Find the coordinates of a point on the line / Bepaal koördinate van 'n punt op die lyn
- Substitute m and the point in / Vervang m en die punt in: $(y - y_A) = m(x - x_A)$

General form / Standaardvorm: $y = a(x + p)^2 + q$

Sketch / Skets

- Turning point / Draaipunt: $(p; q)$
- Form / Vorm: a value / a waarde
- x -intercepts / x -afsnitte: Let / Stel $y = 0$
- y -intercept / y -afsnit: Let / Stel $x = 0$

Sketch using transformations / Skets mbv transformasies:

- $y = a(x - p)^2 + q$
- Draw the graph / Skets die grafiek van: $y = ax^2$
- Shift graph q units in the vertical direction / Skuff grafiek q eenhede vertikaal
- Shift graph $(-p)$ units in the horizontal direction / Skuff grafiek $(-p)$ eenhede horisontaal

Determining the equation / Bepaling van die vergelyking

- Turning point + any other point / Draaipunt + enige ander punt: $y = a(x - p)^2 + q$
- Two roots + any point / Twee wortels + enige punt: $y = a(x - x_1)(x - x_2)$

Hyperbola / Hiperbool

General form / Algemene vorm: $y = \frac{a}{x}$; if $a > 0$ graph is in the 1st and 3rd quadrants / as $a > 0$ dan is grafiek in 1st en 3rd kwadrante

If graph is not in standard position / Indien grafiek nie in standaard posisie is nie: $y = \frac{a}{(x+p)} + q$; with / met

- a (vertical stretch / vertikale uitrekking)
- p (horizontal shift / horizontale skuif)
- q (vertical shift / vertikale skuif)

Asymptotes of the graph / Asimptote van die grafiek: $y = \frac{a}{(x+p)} + q$

- Horizontal asymptote / Horizontale asimptoot: $y = q$
- Vertical asymptote / Vertikaal asimptoot: $x = -p$

Sketch using a table / Skets grafiek mbv tabel

- Get y alone. / Kry y alleen aan 'n kant.
- Table: Choose x and calculate y . / Tabel: Kies x en bereken y .

Sketch using transformations / Skets mbv transformasies: $y = \frac{a}{(x+p)} + q$

- Draw the graph / Skets grafiek van: $y = \frac{a}{x}$
- Shift graph q units in the vertical direction / Skuif grafiek q eenhede in die vertikale rigting
- Shift graph $(-p)$ units in the horizontal direction / Skuif grafiek $(-p)$ eenhede in die horisontale rigting



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Exponential graph / Eksponensiële grafiek

General form / Algemene vorm: $y = b^x$, $b > 0$

If graph is not in standard position / Indien grafiek nie in standaard posisie is nie: $y = a \cdot b^{(x+p)} + q$, with / met

- a (vertical stretch / vertikale uitrekking)
- p (horizontal shift / horizontale skuff)
- q (vertical shift / vertikale skuff)

Asymptotes of the graph / Asimptote van die grafiek: $y = a \cdot b^{(x+p)} + q$

- Horizontal asymptote / Horizontale asimptoot: $y = q$

Sketch using a table / Skets grafiek mbv tabel

- Get y alone. / Kry y alleen aan 'n kant.
- Table: Choose x and calculate y . / Tabel: Kies x en bereken y .

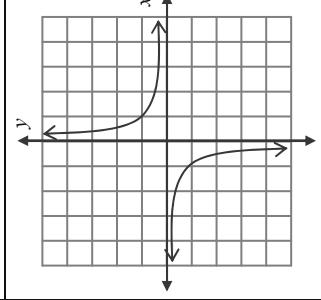
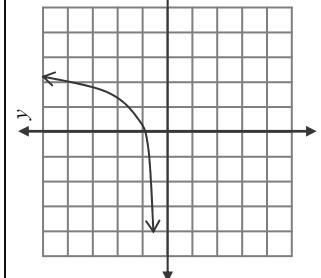
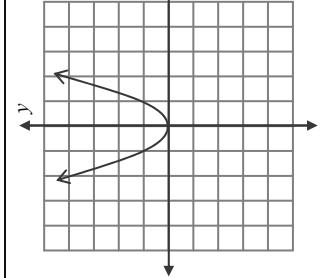
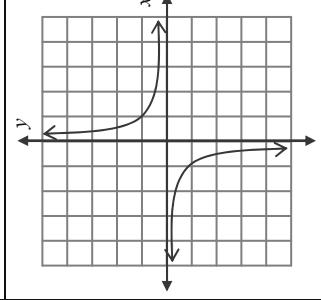
Sketch using transformations / Skets mbv transformasies: $y = a \cdot b^{(x+p)} + q$

- Draw the graph / Skets grafiek van: $y = a \cdot b^x$
- Shift graph q units in the vertical direction / Skuff grafiek q eenhede in die vertikale rigting
- Shift graph $(-p)$ units in the horizontal direction / Skuff grafiek $(-p)$ eenhede in die horizontale rigting

Determining the equation / Bepaling van die vergelyking: $y = a \cdot b^{(x+p)} + q$

- Find horizontal asymptote: If equation is $y = 3$ then $q = 3$ / Bepaal die horizontale asimptoot: As $y = 3$ dan is $q = 3$
- Use y -intercept to find a : If $p = 0$: $a = y$ -intercept / Gebruik y -afsnit om a te bepaal: As $p = 0$ dan is $a = y$ -afsnit
- Substitute any point on the exponential graph temporarily in $y = b^x$ and calculate b . / Vervang enige punt tydelik in die vergelyking om b te bepaal.

Properties of graphs / Eienskappe van grafieke

Linear function	Quadratic	Exponential	Hyperbola
$y = x$	$y = x^2$	$y = b^x$	$y = \frac{1}{x}$
			
domain	\mathbb{R}	\mathbb{R}	$(-\infty; 0) \cup (0; +\infty)$
range	\mathbb{R}	$[0; +\infty)$	$(-\infty; 0) \cup (0; +\infty)$
x -intercepts	0	0	-
y -intercepts	0	0	1
turning points	-	$(0; 0)$ is a minimum	-
asymptotes	-	-	$y = 0$
symmetry	no symmetry	$x = 0$	$y = -x$
intervals on which the function increases	\mathbb{R}	$(0; +\infty)$	\mathbb{R}
intervals on which the function decreases	-	$(-\infty; 0)$	$(-\infty; 0) \cup (0; +\infty)$



Worksheet / Werkkaart 7

1. Given/Gegee: $f(x) = 2 \cos(x + 30^\circ)$
Find the following properties of f .
 - a) maximum value / maksimum waarde
 - b) period / periode
 - c) range / waardeversameling
 - d) amplitude
2. Given/Gegee: $f(x) = 3\tan x - 2$.
Find the following properties of f / Bepaal die volgende eienskappe van f .
 - a) y -intercept / y -afsnitte
 - b) general equation of the asymptotes
 - c) algemene vergelyking van die asymptote
 - d) period / periode
 - e) range / waardeversameling
3. Sketch the graph of/Skets die grafiek van $f(x) = (x - 3)^2 + 2$.
Use your graph to find the following properties about f .
Gebruik jou grafiek om die volgende eienskappe van f te bepaal:
 - a) y -intercept / y -afsnitte
 - b) x -intercepts / x -afsnitte
 - c) minimum value / minimum waarde
 - d) domain / definisie versameling
 - e) range / waardeversameling
 - f) line(s) of symmetry / lyne van simmetrie
4. Draw the graph of / Skets die grafiek van: $f(x) = 3^x$
 - a) Use a transformation to draw the graph of $y = 3^{-x}$ on the same set of axes.
Gebruik 'n transformasie om die grafiek van $y = 3^{-x}$ op dieselfde assestelsel te teken.
 - b) Give the equations of the line of symmetry between the two graphs.
Gee die vergelyking van die simmetriee-as van die twee grafieke.
5. Given/Gegee: $g(x) = x^2$ and/en $f(x) = 3(x - 2)^2 - 1$
 - a) Graph the function of $g(x)$ / Skets die grafiek van $g(x)$.
 - b) Describe a series of transformations needed to obtain the graph of $f(x) = 3(x - 2)^2 - 1$ from the graph of $g(x) = x^2$.
Beskryf 'n reeks transformasies wat gebruik kan word om die grafiek van $f(x) = 3 - (x + 4)^2$ vanaf die grafiek $g(x) = x^2$ te verkry.
 - c) Graph the function $f(x)$ by starting with the graph of $g(x)$ by using transformations.
Skets die funksie $f(x)$ deur met die grafiek van $g(x)$ te begin en deur transformasies te gebruik.
 - d) Give the domain of $f(x)$
 - e) Gee die definisie versameling van $f(x)$.
 - f) Give the range of $f(x)$
 - g) Gee die waardeversameling van $f(x)$.

Worksheet / Werkkaart 8

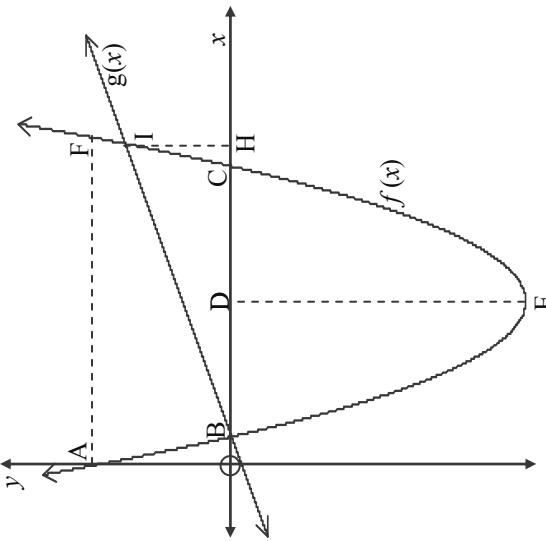
1. The monthly electricity cost depends on the number of units used during the month. Ann found that in April her electricity bill was R460 for 420 units and in May the bill was R620 for 720 units.
- Express the monthly cost C in terms of the number of units used x , assuming that a linear relationship gives a suitable model.
 - Use a) to predict the cost of 2000 units per month.
 - Draw the graph of the linear equation.
 - What does the slope of the line represent?
 - What does the y-intercept of the graph represent?
2. Draw the graphs of $y = x^2 + x - 6$ and $y = 2x + 1$ and use the graphs to answer the following questions:
- If $f(x) = \frac{x^2 + x - 6}{2x + 1}$, then complete:
- For which value(s) of x is $f(x) = 0$
 - For which value(s) of x is $f(x) \leq 0$
 - For which value(s) of x is $f(x) \geq 0$
 - For which value(s) of x does $f(x)$ not exist?
3. Given: $f(x) = x^2 - x - 6$ and $g(x) = x + 2$
- Sketch the graphs of $g(x)$ and $f(x)$ on the same set of axes.
 - Use your graphs to solve the equation $x^2 - 2x - 8 = 0$ to los.

1. Die maandelikse koste van elektrisiteit hang af van die aantal eenhede wat gebruik word gedurende die maand. Ann se koste in April maand was R460 vir 420 eenhede wat sy gebruik het en in Mei was dit R620 vir 720 eenhede wat sy gebruik het.
- Druk die maandelikse koste C uit in terme van die aantal eenhede x wat gebruik is, as aanvaar word dat die model deur 'n lineêre verwantskap voorgestel kan word.
 - Gebruik die antwoord in a) en voorspel die koste as 2000 eenhede per maand gebruik word.
 - Stel die lineêre vergelyking grafies voor.
 - Wat stel die gradiënt van die lyn voor?
 - Wat stel die y-afsnit van die grafiek voor?
2. Teken die grafieke van $y = x^2 + x - 6$ en $y = 2x + 1$ en gebruik hulle om die volgende vrae te beantwoord:
- $$\text{As } f(x) = \frac{x^2 + x - 6}{2x + 1} \text{ voltooi:}$$
- Vir watter waardes van x is $f(x) = 0$
 - Vir watter waardes van x is $f(x) \leq 0$
 - Vir watter waardes van x is $f(x) \geq 0$
 - Vir watter waardes van x sal $f(x)$ ongedefinieer wees?
3. Gegee: $g(x) = x + 2$ & $f(x) = x^2 - x - 6$
- Skets die grafieke van $g(x)$ en $f(x)$ op dieselfde assestelsel.
 - Gebruik jou grafieke om die vergelyking $x^2 - 2x - 8 = 0$ te los.



Worksheet / Werkkaart 9

1. In the figure the graphs of the following functions are shown:
 In die figuur word die grafiese van die volgende funksies
 aangetoon:

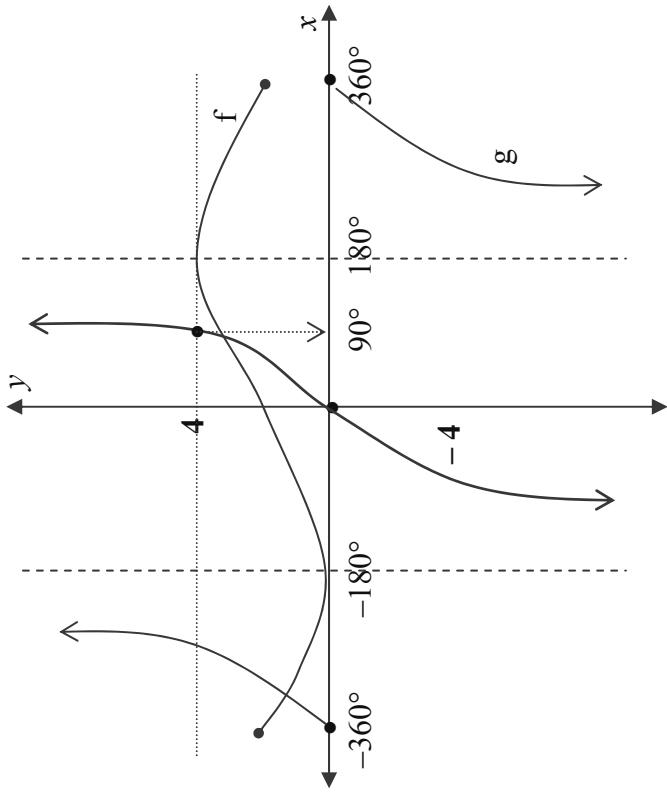


E is the turning point of f .
 $f(x) = x^2 - 10x + 9$ and/or $g(x) = 2x - 2$

- a) Determine the coordinates of the following points:
 i) A
 ii) B and/or C
 b) Determine the lengths of / Bepaal die lengtes van:
 i) OD
 ii) DE

- iii) AF
 iv) HI

2. The figure shows the graphs of two trigonometric functions,
 $f(x)$ and $g(x)$. Determine the equation of $f(x)$ and $g(x)$.
- Die figuur toon grafiese van twee trigonometriese funksies,
 $f(x)$ en $g(x)$. Bepaal die vergelykings van $f(x)$ en $g(x)$.



Differential Calculus / Differensiaalrekening

Average decline / Gemiddelde helling: $\frac{f(x+h) - f(x)}{h}$ (Average speed / Gemiddelde spoed)

The incline in a point / Die helling in 'n punt: $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

Notation / Notasie: $f'(x)$ or $D_x(f)$ or $D_x(f(x))$ or $\frac{dy}{dx}$ or $\frac{d}{dx}(f(x))$

Rate of change / Veranderingsstempo: $f'(x)$

Remember $s(t) \Rightarrow$ distance & $s'(t) \Rightarrow$ speed & $s''(t) \Rightarrow$ speed

Wordproblems / Woordsomme:

- Draw a sketch / Teken 'n skets
- Draw up an equation, based on the problem, eg. $f(x)$. / Stel 'n vergelyking op, gebaseer op die probleem, bv. $f(x)$.
- Determine / Bepaal $f'(x)$
- Put $f'(x) = 0$, to deliver a maximum or minimum value for x / Stel $f'(x) = 0$ om 'n maksimum of minimum waarde vir x te lewer.
- Substitute x in $f(x)$ to get the maximum or minimum value. / Vervang x in $f(x)$ om die maksimum of minimum waarde te kry

Sketches of polynomials of the third degree / Sketse van derdegraadse polinome:

General equation / Algemene vergelyking: $y = ax^3 + bx^2 + cx + d$

- X-intercepts / X-afsnitte: $f(x) = 0$
- Factorise through grouping / factor theorem
- Factoriseer deur groepering / faktorstelling
- Y-intercept / Y-afsnit: Let / Stel $x = 0$
- Stationary points / Stationäre punte: Put / Stel $f'(x) = 0$
 - Solve for x / Los op vir x .
 - Find $f(x)$ for abovementioned x -values.
 - Vind $f(x)$ vir bogenoemde x -waardes.



Worksheet / Werkkaart 10

1. Given / Gegee: $f(x) = 3x - 2x^2$
 Determine the average rate of change of the function between $x = -2$ and $x = 3$.
 Bepaal die gemiddelde veranderingstempo van die funksie tussen $x = -2$ en $x = 3$.

2. Given / Gegee: $f(x) = \frac{2x}{x-1}$

- a) Determine the average rate of change of the function between $x = -1$ and $x = 2$.
 Bepaal die gemiddelde veranderingstempo van die funksie tussen $x = -1$ en $x = 2$.
- b) Determine the average rate of change of the function between $x = a$ and $x = a + h$.
 Bepaal die gemiddelde veranderingstempo van die funksie tussen $x = a$ en $x = a + h$.

3. If $f(x) = -4x^2$, determine $f'(x)$ from first principles.
 As $f(x) = -4x^2$, bepaal $f'(x)$ vanaf eerste beginsels.

4. Determine $f'(x)$ / Bepaal $f'(x)$:

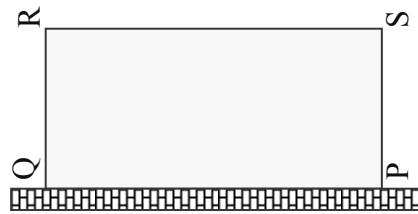
- a) $f(x) = (x - 1)(x^2 + 2)$
 b) $f(x) = 2x - \frac{4}{x^2} + 3$
 c) $f(x) = 3x^{-4} + \sqrt{x}$
 d) $f(x) = 3x^3 + 2x^2 - 4x + 6$
 e) $f(x) = \sqrt{x} + (x + 3)^2$

5. a) Find the equations of the tangents to the parabola $y = x^2 - 2x - 3$ at the points where $x = 0$ and $x = 2$ respectively.
 Bepaal die vergelykings van die raaklyne aan die parabol $y = x^2 - 2x - 3$ by die punte waar $x = 0$ en $x = 2$ respektief is.
- b) Find the point of intersection of these two tangents.
 Bepaal die snypunt van hierdie twee raaklyne.
6. Sketch the following graphs / Skets die volgende grafieke:
 a) $f(x) = x^3 - 4x^2 - 11x + 30$
 b) $f(x) = x^3 - 7x^2 + 15x - 9$
 c) $f(x) = x^3 - x^2 - 8x + 12$
7. A cold drink company wants to optimise its manufacturing cost. They want to use a closed cylindrical can that holds 350 ml of cold drink to sell their products. Calculate the height and radius that will minimise the amount of material needed to manufacture a can.

Worksheet / Werkkaart 11

1. A biologist states that when a specific kind of anti-bacterium is added to a culture of bacteria, the number of bacteria present will be given by the formula $B(t) = -5t^2 + 60t + 1200$ where $B(t)$ represents in millions the number of bacteria on moment t.
- ’n Bioloog beweer dat wanneer ’n sekere soort antibakterie tot ’n kultuur van bakterieë gevoeg word, die getal bakterieë teenwoordig gegee sal word deur die formule $B(t) = -5t^2 + 60t + 1200$ waar $B(t)$ in miljoene die getal bakterieë op tydstip t , gemit in uur, voorstel.
- How many bacteria were present at the beginning? / Hoeveel bakterieë was aan die begin teenwoordig?
 - Calculate the tempo of change in relation to the time on moment $t = 10$ hours. / Bereken die tempo van verandering met betrekking tot die tyd op tydstip $t = 10$ uur.
 - Was the bacteria population shrinking or growing on moment $t = 10$ hours? / Was die bakterieë bevolking aan die afneem of toename op tydstip $t = 10$ uur?
 - At which moment was the maximum number of bacteria present? / Op watter tydstip was die maksimum getal bakterieë teenwoordig?
 - After how many hours did the number of bacteria start decreasing? / Na hoeveel uur het die getal bakterieë begin afneem?

2. The humidity (H) of air in relation to the temperature (t in $^{\circ}\text{C}$) is given by the formula $H = -t^3 + 24t^2 - 84t + 80$
- Die humiditeit (H) van lug met betrekking tot die temperatuur (t in $^{\circ}\text{C}$) word gegee deur die formule $H = -t^3 + 24t^2 - 84t + 80$
- Determine the humidity at temperature $0 ^{\circ}\text{C}$ / Bepaal die humiditeit by temperatuur $0 ^{\circ}\text{C}$.
 - At which temperature is the humidity $0?$ / By watter temperatuur is die humiditeit $0?$
 - At which temperature does the humidity reach a maximum in the interval $[0 ^{\circ}; 20 ^{\circ}]?$ / By watter temperatuur bereik die humiditeit ’n maksimum in die interval $[0 ^{\circ}; 20 ^{\circ}]?$
 - Sketch the graph of H with relation to t in the interval $[0 ^{\circ}; 20 ^{\circ}].$ / Skets die grafiek van H met betrekking tot t in die interval $[0 ^{\circ}; 20 ^{\circ}]$.



3. A rectangular camp PQRS that must be fenced in by 72 cm of wire netting. PQ is an existing wall. PS, SR and QR must be fenced in. Calculate the maximum area of the camp.
- Die meegaande figuur toon ’n reghoekige kamp PQRS wat omhein moet word deur 72 cm ogiesdraad. PQ is ’n bestaande muur. PS, SR en QR moet toegekamp word. Bereken die maksimum oppervlakte van die kamp.

Paper 2/ Vraestel 2

Coordinate geometry / Analyiese meetkunde

Trigonometry / Trigonometrie

Data handling / Data hantering

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Volume and surface area / Volume en buiteoppervlakte

Cylinder

- Volume of a cylinder = $\pi r^2 h$
- Surface area (closed cylinder) = $2\pi r^2 + 2\pi r h$

Prism

- Volume of prism = (area of base) × height
- Surface area = $A + ph = ab + h(a+b)$

Cone

- Volume of cone = $\frac{1}{3} \pi r^2 h$
- Surface area of closed cone = $\pi r^2 + \pi r H$

Pyramid

- Volume of pyramid = $\frac{1}{3} Ah$
- Surface area of pyramid = $A + \frac{1}{2} pH$

Sphere

- Volume of sphere = $\frac{4}{3} \pi r^3$
- Surface area of sphere = $4\pi r^2$



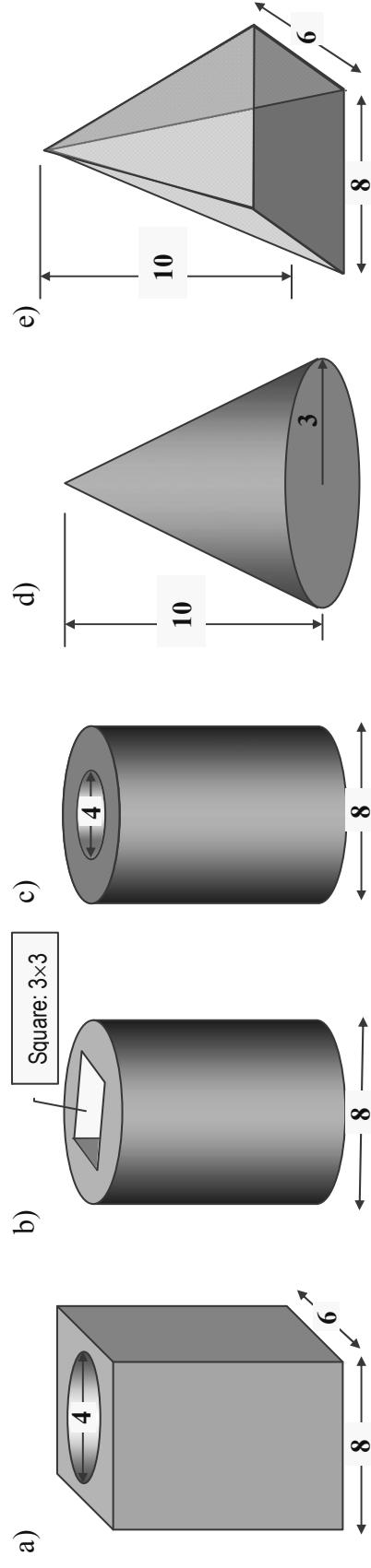
Worksheet / Werkkaart 12

1. Calculate the number of diagonals in a polygon with 12 sides / Bepaal hoeveel diagonale 'n veelhoek met 12 sye het.
2. Complete the following table / Voltooi die volgende tabel:

Number of sides of a polygon / Aantal sye van veelhoek	3	4	5	100	n
Sum of interior angles / Som van die binnehoeke	180°	360°			
In the case of a regular polygon find the measure of each angle. In die geval van 'n reëlmataige veelhoek bepaal die grootte van elke hoek	60°	90°			

3. Determine the measure of each of the twelve angles of a regular dodecagon / Bepaal die mates van die binnehoeke van 'n reëlmataige twaalfhoek.

4. Calculate the surface area of each solid correct to two decimal places. Lengths are in cm and the height of each of the prisms and pyramids is 10 cm.
Bereken die buite oppervlakte van elk van die volgende vaste liggame korrek tot twee desimale syfers. Lengtes is in cm en die hoogtes van alle prismas en piramide is 10cm.



Analytical geometry / Analytiese meetkunde

Formulae / Formules

Distance between points A and B / Afstand tussen twee punte A en B: $AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$

Midpoint M($x_M ; y_M$) of line segment AB / Midelpunt M($x_M ; y_M$) van AB: $x_M = \frac{x_A + x_B}{2}$ and $y_M = \frac{y_A + y_B}{2}$

Gradient of a line / Helling (gradiënt): $m_{AB} = \frac{y_A - y_B}{x_A - x_B}$

Angle of inclination / Inklinasie hoek: $\tan \theta = m$

Equation of a straight line / Vergelyking van 'n reguitlyn: $y = mx + c$ of/or $y - y_A = m(x - x_A)$

Circle with centre ($a; b$) and radius r / Sirkel met middelpunt ($a; b$) & radius r : $(x - a)^2 + (y - b)^2 = r^2$

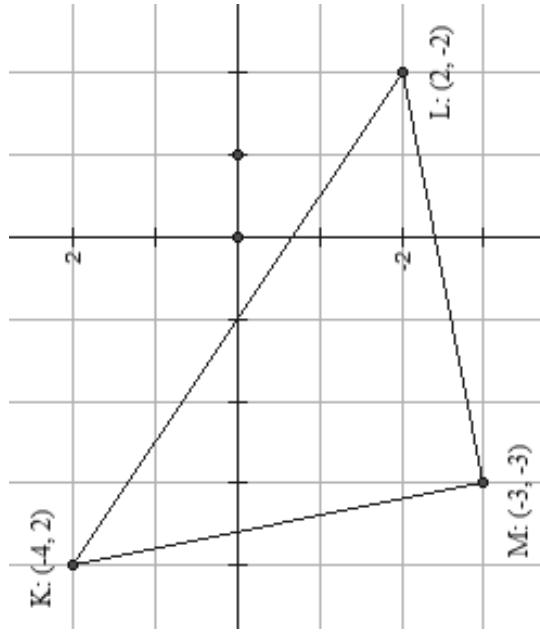


Worksheet / Werkkaart 13

1. A $(-5; 6)$, B $(t; -2)$ and C $(-7; 2)$ are three points in a Cartesian plane. Calculate
 - a) the co-ordinates of the midpoint of AC
 - b) the value(s) of t if $AB = BC$
 2. P $(-6; 2)$ and Q $(q; 6)$ are points on the circumference of a circle with the origin as centre. Determine
 - a) the equation of the circle
 - b) the value(s) of q
 - c) the size of the angle between the line PQ and the positive x -axis if $q = 2$.
 3. B $(3; 2)$ and C $(3; -6)$ are points on the circumference of a circle with centre Q and with equation $x^2 + y^2 - 2x + 4y - 15 = 0$.
 - a) Determine the coordinates of the centre of the circle and the length of the radius.
 - b) Determine the equation of the tangent to the circle at C.
 4. A $(-4; 5)$, B $(-6; -6)$ and C $(3; -2)$ are the vertices of $\triangle ABC$. Calculate the measure of angle $B\hat{C}A$.
 5. A circle with centre M $(4; 2)$ passes through the point A $(8; -2)$. Determine
 - a) the equation of the circle in the form $x^2 + ax + y^2 + by + c = 0$
 - b) the co-ordinates of the y-intercepts of the circle
 6. What is the radius of the circle with equation $9x^2 = 4 - 9y^2$?
1. A $(-5; 6)$, B $(t; -2)$ en C $(-7; 2)$ is drie punte in 'n Cartesiese vlak. Bereken
 - a) die koördinate van die middelpunt van AC
 - b) die waarde(s) van t as $AB = BC$
 2. P $(-6; 2)$ en Q $(q; 6)$ is punte op die omtrek van 'n sirkel met die oorsprong as middelpunt. Bepaal
 - a) die vergelyking van die sirkel
 - b) die waarde(s) van q
 - c) die grootte van die hoek wat die lyn PQ met die positiewe x-as maak as $q = 2$.
 3. B $(3; 2)$ en C $(3; -6)$ is punte op die omtrek van 'n sirkel met middelpunt Q en met vergelyking $x^2 + y^2 - 2x + 4y - 15 = 0$.
 - a) Bepaal die koördinate van die middelpunt van die sirkel en die radius van die sirkel.
 - b) Bepaal die vergelyking van die raaklyn aan die sirkel by C.
 4. A $(-4; 5)$, B $(-6; -6)$ en C $(3; -2)$ is die hoekpunte van $\triangle ABC$. Bereken die grootte van hoek $B\hat{C}A$.
 5. 'n Sirkel met middelpunt M $(4; 2)$ gaan deur die punt A $(8; -2)$. Bepaal
 - a) die vergelyking van die sirkel in vorm $x^2 + ax + y^2 + by + c = 0$
 - b) die koördinate van die y-afsnitte van die sirkel.
 6. Wat is radius van die sirkel met vergelyking $9x^2 = 4 - 9y^2$?

Worksheet / Werkkaart 14

1. Determine the co-ordinates of the centre and the radius of the circle with equation $x^2 - 10x + y^2 + y = 0$
 2. AB and DE are two parallel lines. The equation of AB is $3y - 2x = 15$ and the co-ordinates D and E are D(0 ; -1) and E(e ; -3) respectively. Determine
 - a) the gradient of the straight line AB
 - b) the value(s) of e
 - c) the equation of the straight line through the point C(-3 ; 8) which is perpendicular to AB, in the form $ax + by + c = 0$.
 3. K(-4; 2), L(2; -2) and M(-3; -3) are vertices of $\triangle KLM$ in a Cartesian plane.
 - a) Calculate the perimeter of $\triangle KLM$.
 - b) Calculate the inclination angle of LM.
 - c) Determine the co-ordinates the midpoint of segment KM.
 - d) Determine the equation of the perpendicular bisector of KM.
1. Bepaal die koördinate van die middelpunt en radius van die sirkel met vergelyking $x^2 - 10x + y^2 + y = 0$
2. AB en DE is twee ewewydige lyne. Die vergelyking van AB is $3y - 2x = 15$ en die koördinate van D en E is onderskeidelik D(0 ; -1) en E(e ; -3). Bepaal
 - a) die gradiënt van die reguitlyn AB
 - b) die waarde(s) van e
 - c) die vergelyking van die reguitlyn deur die punt C(-3 ; 8) wat loodreg is op AB, in die vorm $ax + by + c = 0$.
3. K(-4; 2), L(2; -2) en M(-3; -3) is hoekpunte van $\triangle KLM$ op 'n Kartesiese vlak.
- a) Bereken die omtrek van $\triangle KLM$
 - b) Bereken die inklinasiehoek van LM
 - c) Bepaal die koördinate van die middelpunt van lynstuk KM.
 - d) Bepaal die vergelyking van die loodregte verdeler van KM.



Trigonometry / Trigonometrie

Definitions

$$\sin \theta = \frac{y}{r} = \frac{t}{s} \quad \cos \theta = \frac{x}{r} = \frac{a}{s} \quad \tan \theta = \frac{y}{x} = \frac{t}{a}$$

Simplifying with reducing formulae:

Step 1 Make negative \angle 's positive then $\cos x$ is positive, the rest is negative / Maak negatiewe \angle e positief dan $\cos x$ is positief, en die res negatief
 e.g. $\sin(-300^\circ) = -\sin 300^\circ$ & $\cos(-250^\circ) = +\cos 250^\circ$

Step 2 Make angle greater than 360° / smaller than 360° by subtracting any multiple of 360° , then nothing changes. / Maak hoeke groter as 360° kleiner as 360° deur veelyoude van 360° af te trek, maar geen tekens verander nie.
 e.g. $\cos 1000^\circ = \cos 280^\circ$

Step 3 Make all \angle 's acute angles ($< 90^\circ$). Discard multiples of $180^\circ \pm$ or $360^\circ \pm$ and determine the sign by means of CAST table. Maak alle \angle e skerphoeke ($< 90^\circ$), 'gooi $180^\circ \pm$ of $360^\circ \pm$ weg' en bepaal die teken mbv die CAST-tabel:
 e.g. $\cos 150^\circ = \cos(180^\circ - 30^\circ) = -\cos 30^\circ$

Step 4 For further simplification or if there is $(90^\circ - \beta)$, then use cofunctions / Vir verdere vereenvoudiging of as daar 'n $(90^\circ - \beta)$, gebruik ko-funksies

Proof of identities (LHS & RHS individually) / Bewys van identiteite (LK & RK apart)

- 1) Choose most difficult side / Kies moeilikste kant
- 2) Look for square identities / Kyk vir vierkant identiteite (Kyk vir '1' en kwadrate)
- 3) Everything to sin x and cos x / Alles na sin x & cos x
- 4) Fractions: Get LCM / Breuke: Kry KGV
- 5) Factorise or simplify / Faktoriseer of vereenvoudig
- 6) Emergency plan: If $\cos x - 1$, then multiply above and below by $(\cos x + 1)$ / Noodplan: Indien $(\cos x - 1)$ maal dan bo en onder: $(\cos x + 1)$
- 7) See for which values of x the expression is not defined / Kyk vir watter waardes van x is die uitdrukking nie gedefinieer nie

Identities / Identiteite

$$\tan x = \frac{\sin x}{\cos x} \quad \sin^2 x + \cos^2 x = 1 \quad \text{Cofunction } \sin(90^\circ \pm x) = \cos x \quad \cos(90^\circ + x) = -\sin x$$

Compound & double angles / Saamgestelde- en dubbelhoede

Compound angles / Saamgestelde hoeke

- $\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$
- $\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$
- $\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$
- $\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$

Double angles / Dubbelhoeke

- $\sin 2A = 2\sin A \cdot \cos A$
- $\cos 2A = \cos^2 A - \sin^2 A$
- $\cos 2A = 2\cos^2 A - 1$
- $\cos 2A = 1 - 2\sin^2 A$



Worksheet / Werkkaart 15

1. Given/Gegee: $f(x) = \sin 2x$ and/en $g(x) = \sin(45^\circ + x)$
- Sketch the graphs of f and g on the same set of axes for $x \in [-180^\circ, 180^\circ]$. Clearly show the co-ordinates of the intercepts with the axes and of all turning points.
Skets die grafieke van f en g op dieselfde assestelsel vir $x \in [-180^\circ, 180^\circ]$. Dui die koördinate van die afsnitte op die asse en die draaipunte van die grafieke duidelik aan.
 - Write down the period of f and g
Skryf die periode neer van f en g

2. a) Sketch the graphs of f , g and h for the interval $[-180^\circ, 180^\circ]$, where: / Skets die grafieke van f , g en h vir die interval $[-180^\circ, 180^\circ]$, waar: $f(x) = 2 \cos x$; $g(x) = 2 + \cos x$; $h(x) = \cos 2x$
- For which value(s) of x will $\cos 2x = 0$?
Vir watter waarde(s) van x sal $\cos 2x = 0$?

- Use your graphs to determine the greatest value of the expression $2 - \cos x$. Indicate clearly where you have read your answer, again using capital letters.
Gebruik u grafiek om die grootste waarde van die uitdrukking $2 - \cos x$ te bepaal. Toon duidelik aan waar u die antwoord afgelê het. Gebruik weer eens hoofletters daarvoor.

- Sketch, on the same set of axes, the graphs of / Skets op dieselfde assestelsel, die grafieke van:

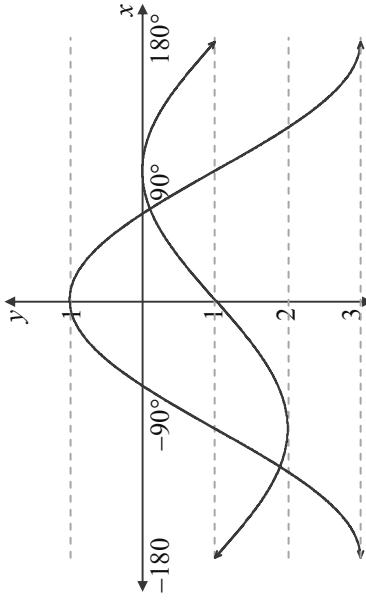
$$f = \{(x; y) / y = \frac{1}{2}\tan x; -180^\circ \leq x \leq 180^\circ\}$$
 and/en

$$g = \{(x; y) / y = \tan \frac{1}{2}x; -180^\circ \leq x \leq 180^\circ\}$$
 - What is the period of f ? / Wat is die periode van f ?
 - For which value(s) of x will $f(x) = g(x)$?
Vir watter waarde(s) van x sal $f(x) = g(x)$?
 - For which value(s) of x will $f(x) > g(x)$?
Vir watter waarde(s) van x sal $f(x) > g(x)$?

4. In the figure, the graphs of two functions / In die figuur word die grafieke van die twee funksies:

$$f = \{(x; y) / y = a \cos x + k; -180^\circ \leq x \leq 180^\circ\}$$
 and/en

$$g = \{(x; y) / y = \sin bx + m; -180^\circ \leq x \leq 180^\circ\}$$



Determine the values of a , b , k , and m . / Bepaal die waardes van a , b , k en m .

Worksheet / Werkkaart 16

1. Simplify, without using a calculator, the following expressions:
(Show ALL the calculations.)

Vereenvoudig, sonder die gebruik van 'n sakrekenaar, die volgende uitdrukings (Toon AL die berekeninge):

a)
$$\frac{\tan(360^\circ - x) \cdot \cos(180^\circ + x)}{\cos(90^\circ + x)}$$

b)
$$\frac{\cos 330^\circ \cdot \tan 135^\circ}{\sin 570^\circ \cdot \cos(-60^\circ)}$$

c)
$$\frac{\tan 240^\circ \cdot \cos 150^\circ - \tan 225^\circ + 4 \sin 90^\circ}{\cos(-300^\circ)}$$

d)
$$\tan 30^\circ \cdot \sin 240^\circ + \cos 180^\circ - \cos^2 135^\circ$$

e)
$$\frac{3 \cos(90^\circ - x) \cdot \sin(360^\circ + x) + \cos^2(360^\circ + x) \cdot \tan 150^\circ}{\sin(-90^\circ) \cdot \tan(-120^\circ)}$$

f)
$$\sin^2 x \left[\frac{1}{1 + \cos x} + \frac{1}{1 - \cos x} \right]$$

2. Determine, without using a calculator, the value of the following in terms of t , if $\sin 17^\circ = t$ / Bepaal, sonder die gebruik van 'n sakrekenaar, die waarde van die volgende in terme van t indien $\sin 17^\circ = t$:

- a) $\sin 523^\circ$
b) $\cos 73^\circ$
c) $\tan(-17^\circ)$

3. If $x \in [0^\circ; 180^\circ]$ solve the following equations / As $x \in [0^\circ; 180^\circ]$ los die volgende vergelykings op:
- $\cos A = -\frac{\sqrt{3}}{2}$
 - $\sin^2 x = 1 - \cos^2 x$
 - $\cos(2x - 72^\circ) = \sin(x + 23^\circ)$
4. Solve for x if / Los op vir x , indien:
- $5\cos 3x + 4 = 0$ and/en $x \in [0^\circ; 360^\circ]$.
 - $2\sin(A - 20^\circ) = -0,428$; $A \in [-90^\circ; 270^\circ]$
5. If/As $4 \cdot \cos x = 2\sqrt{3}$, $\sqrt{2} \sin y + 1 = 0$ and/en $x, y \in [180^\circ; 360^\circ]$ calculate without the use of a calculator the value of / bereken, sonder die gebruik van 'n sakrekenaar die waarde van
- $x + y$
 - $\tan x$
6. Determine the general solution of / Bepaal die algemene oplossing van:
- $2 \cdot \cos x \cdot \sin x - \cos x = 0$
 - $7 \cos \theta - 2 \sin^2 \theta + 5 = 0$
7. Prove the identity / Bewys die identiteit:
- $$\frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} = \frac{4 \tan x}{\cos x}$$
8. If x and y are complementary angles and $13 \cos x - 5 = 0$, calculate, without using a calculator, the value of $\tan x + \cos y$
As x en y komplementêre hoekte is en $13 \cos x - 5 = 0$, bereken,
sonder om 'n sakrekenaar te gebruik die waarde van $\tan x + \cos y$



Solving triangles / Oplos van driehoeke

Sin rule / reël

If \angle and side opposite each other are known + any other angle or side (i.e. $\angle\angle\text{s}$ or $\text{ss}\angle$)
 Indien \angle en sy regoor mekaar bekend is + enige ander hoek of sy m.a.w. $\angle\angle\text{s}$ of $\text{ss}\angle$)

If you want to calculate \angle / Indien \angle gevra word: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

If you want to calculate a side / Indien sy gevra word: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cos rule / reël

If no \angle and side opposite each other are known (i.e. sss or $\text{s}\angle\text{s}$)
 Indien geen \angle en sy regoor mekaar nie bekend is nie (m.a.w. sss of $\text{s}\angle\text{s}$)

If side / Indien sy gevra: $a^2 = b^2 + c^2 - 2bc \cos A$

If \angle / As \angle gevra word: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Area rule / Opp reël

If $90^\circ \angle$ / Indien $90^\circ \angle$ in driehoek: Area = $\frac{1}{2} b \cdot h$

Determine $\text{s}\angle\text{s}$: Area $\Delta ABC = \frac{1}{2} b \cdot c \cdot \sin A$

Worksheet / Werkkaart 17

1. If in $\triangle ABC$, $c = 8$ units, $b = 7$ units and $\hat{B} = 60^\circ$, calculate the following without using a calculator.

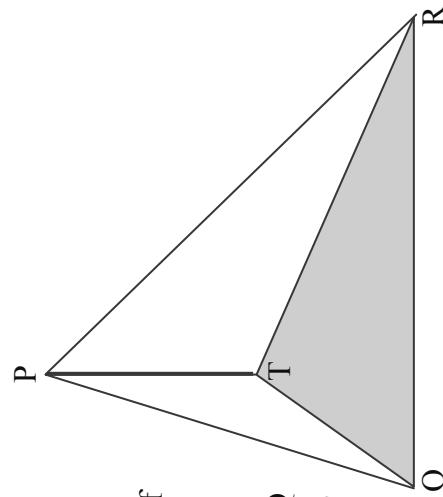
As in $\triangle ABC$, $c = 8$ eenhede, $b = 7$ eenhede en $\hat{B} = 60^\circ$, bereken die volgende sonder die gebruik van die sakrekenaar:

- a) the value of a / die waarde van a .

- b) the area of $\triangle ABC$ for the greatest value of a .
die oppervlakte van $\triangle ABC$ vir die grootste waarde van a .

2. In the figure Q, T and R are points in the same horizontal plane such that $TQ = TR = y$ and TP represents a vertical pole positioned at T.

In die figuur is Q, T en R punte in dieselfde horisontale vlak sodat $TQ = TR = y$ en TP stel 'n vertikale paal voor.



- a) Prove that / Bewys dat:

$$PQ = PR.$$

- b) If the angle of elevation of P from Q is α and

$$\hat{P}RQ = \beta /$$

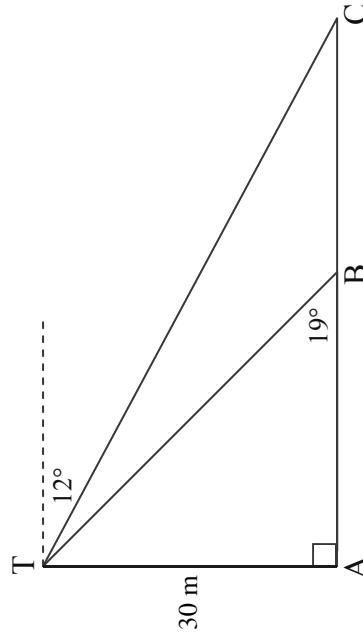
As die hoogtehoek van P vanaf Q gelyk is aan α en $P\hat{R}Q = \beta$

- i) express PQ in terms of y and a trigonometric value of α / Q druk PQ uit in terme van y en 'n trigonometriese funksie van α .

- ii) Prove that / Bewys dat: $QR = \frac{y \sin 2\beta}{\sin \beta \cdot \cos \alpha}$

3. The diagram below is a representation of a tree (TA) with height of 30 m and two observers K and L on the ground. The angle of depression from T to person C is 12° . The angle of elevation from person B to the top of the tree is 19° .

Die diagram hieronder is 'n voorstelling van 'n 30 m hoë boom (TA) en twee persone op die grond. Die dieptehoek van T na persoon C is 12° . Die hoogtehoek van B na die top van die boom 19° .



- a) Calculate the size of \hat{C} / Bereken die grootte van \hat{C} .
b) Calculate the length of TB / Bereken die lengte van TB.
c) Hence calculate the distance between the two observers B and C / Bereken vervolgens die afstand tussen die twee persone B en C



Data handling / Data Hantering

Measures of Central Tendency / Bepalers van sentrale neiginge

- Mean (Average) / Gemiddeld
 - The sum of a set of data divided by the number of data. / Die som van die gegewe data gedeel deur die aantal data: $\bar{x} = \frac{\sum x}{n}$
- Median (middle value) / Mediaan (middelwaarde)
 - The value halfway through an ordered data set. / Die waarde halfpad in die geordende data versameling.
- Mode (most frequent value) / Modus
 - Value that appears the most. / Waarde wat die meeste voorkom.

Measures of dispersion (variability) / Maat van dataverspreiding

- Range (Largest value – smallest value) / Omvang (Grootste waarde – kleinste waarde)
- Percentiles (% of total frequency $\leq x$) / Persentiel (% van die totale frekwensie $\leq x$)
- Percentiles divide the data in 100 groups containing the same number of observations. / Persentiele deel die data in 100 groepe wat die selfde aantal observasies bevat.
- Variance / Variansie:
$$\frac{\sum (x - \bar{x})^2}{n}$$
- Standard deviation / Standaard afwyking:
$$\sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

Worksheet / Werkkaart 18

- Given data / Gegewe data: 5, 9, 7, 8, 3, 10, 8, 10, 9, 17, 33
Find the following for the data / Bepaal die volgende vir die data:
 a) mean / gemiddeld
 b) median / mediaan
 c) mode / modus
 d) range / omvang
 e) lower quartile / onderste kwartiel
 f) upper quartile / boonste kwartiel
 g) interquartile / inter-kwartiel
 h) standard deviation / standaard afwyking
 i) variance / variansie
- Given data/Gegewe data: 5, 9, 7, 8, 3, 10, 8, 10, 9, 17, 33
Represent the data using a box and whisker diagram / Stel die data voor deur van 'n mond-en-snordiagram gebruik te maak.
- The frequency table represents the marks in terms of %, obtained by a group of Grade 12 learners in a Mathematics examination. Die frekvensietabel verteenwoordig die punte in terme van %, deur 'n groep Graad 12-leerders in 'n Wiskunde-eksamen behaal.

Test scores/ Toetspunte	Frequency/ Frekvensie	Cumulative frequency/ Kumulatiewe frekvensie
1-20	3	
21-40	5	
41-60	9	
61-80	6	
81-100	2	

- b) Use the table on the diagram sheet to complete the cumulative frequency column / Gebruik die tabel op die diagramvel om die kumulatiewe frekvensie kolom te voltooi.
 c) Draw the ogive for the given data on a grid / Teken die ogief op 'n rooster.
 d) Use the ogive to determine the median mark / Gebruik die ogief om die mediaanpunt te bepaal.
 - The table represents the number of cars sold by a car manufacturer from 2003 to 2008:
Die onderstaande tabel stel die aantal motors wat verkoop is deur 'n motorvervaardiger 2003 tot 2008, voor:
- | Year / Jaar | Cars sold / Motors verkoop |
|-------------|----------------------------|
| 2002 | 1234 |
| 2003 | 1432 |
| 2004 | 1672 |
| 2005 | 1752 |
| 2006 | 2013 |
| 2007 | 2193 |
| 2008 | 2345 |
- a) Draw the scatter plot to represent the above data. / Teken 'n spreidiagram om bogenoemde inligting voor te stel.
 b) Explain whether a linear, quadratic or exponential curve would be a line of best fit for the above-mentioned data. / Verduidelik of 'n lineêre, kwadratiese of eksponensiële kurwe die beste paslyn vir die bogenoemde inligting sal wees.
 c) If the same trend continued, estimate, by using your graph, the number of cars that will be sold in 2010. / Indien dieselfde neiging/patroon voortgaan, skat, deur jou grafiek te gebruik, die aantal motors wat teen 2010 verkoop sal word.



Euclidean Geometry

Theorems and formulae

Theorems

THEOREM	ABBREVIATION
1a) If two straight lines intersect, the sum of any pair of adjacent angles is equal to 180° .	straight \angle
1b) (CONVERSE) If the sum of any pair of adjacent angles is equal to 180° their non-common sides lie on the same line.	adj. \angle s suppl.
2) If two lines intersect, the vertically opposite angles are equal.	vert. opp. \angle s
3a i) If a transversal intersects two parallel lines, then the corresponding angles are equal to one another.	\parallel lines, corr. \angle s
3a ii) If a transversal intersects two parallel lines, then the alternate angles are equal to one another.	\parallel lines, alt. \angle s
3a iii) If a transversal intersects two parallel lines, then the sum of the interior angles on the same side of the transversal is 180° .	\parallel lines, co-int. \angle s
3b i) (CONVERSE) If two lines are intersected by a transversal so that a pair of corresponding angles are equal, then the two lines are parallel.	corr. \angle s equal
3b ii) (CONVERSE) If two lines are intersected by a transversal so that a pair of alternate angles are equal, then the two lines are parallel.	alt. \angle s equal
3b iii) (CONVERSE) If two lines are intersected by a transversal so that a pair of co-interior angles are supplementary, then the two lines are parallel.	co-int. \angle s suppl.
4) Lines that are parallel to the same line are parallel to each other.	\parallel to the same line
5) The exterior angle of a triangle is equal to the sum of the interior opposite angles.	ext. \angle of \triangle
6) The sum of the angles of a triangle is 180° .	sum \angle s \triangle
7i) If two sides and the included angle of one triangle are respectively equal to two sides and the included angle of another triangle, the two triangles are congruent.	s \angle s
7ii) If two angles and a side of one triangle are respectively equal to two angles and the corresponding side of another triangle, the triangles are congruent.	$\angle\angle$ s

THEOREM	ABBREVIATION
7iii) If three sides of one triangle are respectively equal to three sides of another triangle, the triangles are congruent.	sss
7iv) If in two right-angled triangles the hypotenuse and one side of the one are respectively equal to the hypotenuse and one side of the other, the triangles are congruent.	90°ss
8a) In an isosceles triangle the angles opposite the equal sides are equal.	∠s opp. equal sides
8b) (CONVERSE) If two angles of a triangle are equal then the sides opposite them are equal.	sides opp. equal ∠s
9a) The opposite sides of a parallelogram are equal.	opp. sides \parallel^m
9b) (CONVERSE) If the opposite sides of a quadrilateral are equal, it is a parallelogram.	opp. sides quad. equal
10a) The opposite angles of a parallelogram are equal.	opp. ∠s \parallel^m
10b) (CONVERSE) If the opposite angles of a quadrilateral are equal, it is a parallelogram.	opp. ∠s quad. equal
11) Each diagonal bisects the area of the parallelogram.	diagonal bisects \parallel^m
12) A parallelogram and a rectangle on the same base and between the same parallels have equal areas.	same base and height
13) The area of a triangle is equal to one half the area of a rectangle on the same base and between the same parallels.	same base and height
14a) The diagonals of a parallelogram bisect each other.	diagonals \parallel^m
14b) (CONVERSE) If the diagonals of a quadrilateral bisect each other, it is a parallelogram.	diagonals of quad. bisect
15) If a pair of opposite sides of a quadrilateral are equal and parallel, the quadrilateral is a parallelogram.	opp. sides equal and \parallel
16) The diagonals of a rectangle are equal to each other.	diagonals of rectangle
17) The diagonals of a rhombus bisect each other at right angles and bisect the angles of the rhombus.	diagonals of rhombus
18) The line segment joining the mid-points of two sides of a triangle is parallel to the third side, and equal to half the third side.	mid-point theorem



THEOREM**ABBREVIATION**

19) The line drawn through the mid-point of one side of a triangle, parallel to another side, bisects the third side.

line through mid-point of one side \parallel 2nd side

20) If three or more parallel lines cut off equal line segments on one transversal, they cut off equal line segments on any transversal.

\parallel lines make equal intercepts

21a) The square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides.

Pyth.

21b) (CONVERSE) If the square on one side of a triangle is equal to the sum of the squares on the other two sides, then the angle opposite the first side is a right angle.

converse Pyth.

22a) The line segment joining the centre of a circle to the mid-point of a chord is perpendicular to the chord.

line through centre and midpt.

22b) (CONVERSE) The perpendicular drawn from the centre of a circle to a chord bisects the chord.

perp. from centre to chord

22c) (COROLLARY) The perpendicular bisector of a chord passes through the centre of the circle.

perp. bisector of chord

23) The angle that an arc of a circle subtends at the centre of the circle is twice the angle it subtends at any point on the circle.

\angle at centre $= 2 \times \angle$ at \odot^{ce}

24a) The diameter of a circle subtends a right angle at the circumference.

\angle in semi-circle

24b) (CONVERSE) If the angle subtended by a chord at a point on the circle is a right angle, then the chord is a diameter.

chord subt. 90°

24c) (CONVERSE) The circle described on the hypotenuse of a right-angled triangle as a diameter passes through the vertex of the right angle.

\odot with hypotenuse as diameter

25a) Angles in the same segment of a circle are equal.

\angle s in same segm.

25b) (CONVERSE) If a line segment joining two points subtends equal angles at two other points on the same side of the line segment, these four points are concyclic.

line segm. subt. equal \angle s

25c) (COROLLARY) The angles subtended by arcs of equal length in a given circle are equal.

equal arcs, equal \angle s

25d) (COROLLARY) The angles subtended by arcs of equal length in two different circles with equal radii are equal.

equal arcs, equal radii, equal \angle s

26a) The opposite angles of a cyclic quadrilateral are supplementary.

opp. \angle s of cyclic quad.

26b) (CONVERSE) If two opposite angles of a quadrilateral are supplementary, the quadrilateral is a cyclic quadrilateral.

opp. \angle s suppl.

THEOREM**ABBREVIATION**

27a) An exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

ext. \angle of cyclic quad.

27b) (CONVERSE) If an exterior angle of a quadrilateral is equal to the interior opposite angle, then the quadrilateral is cyclic.

ext. \angle = int. opp. \angle

28a) A tangent to a circle is perpendicular to the radius at the point of contact.

tangent \perp radius

28b) (CONVERSE) A line drawn perpendicular to a radius at the point where the radius meets the circle is a tangent to the circle.

line \perp radius

29) Two tangents drawn to a circle from the same point outside the circle are equal in length.

tangents from point outside \odot .

30a) The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment of the circle.

\angle between tangent and chord

30b) (CONVERSE) If an angle between a chord of a circle and a line through the end of that chord is equal to an angle in the alternate segment, that line is a tangent to the circle.

\angle between chord and line
= \angle in alt. segm.

CONCURRENCY

31) The internal bisectors of the angles of a triangle are concurrent. (Point of intersection is the in-centre of the triangle.)

bisectors of \angle s of \triangle

32) The perpendicular bisectors of the sides of a triangle are concurrent. (Point of intersection is the circum-centre of the triangle.)

perp. bisectors of sides of \triangle

33a) The medians of a triangle are concurrent. (Point of intersection is the centroid of the triangle.)

medians of \triangle

33b) (COROLLARY) The medians of a triangle trisect one another.

medians of \triangle

34) The altitudes of a triangle are concurrent. (The point of intersection is the ortho-centre of the triangle.)

altitudes of \triangle

In any triangle, the centroid, orthocentre and circumcentre are collinear. The line through these three points is called the Euler Line.



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RATIO AND PROPORTION

35a) A straight line parallel to one side of a triangle divides the other two sides proportionally.

line \parallel one side of \triangle

35b) (CONVERSE) If a line cuts two sides of a triangle so as to divide them in the same ratio, then that line is parallel to the third side.

line divides two sides of \triangle prop.

36a) If two triangles are equiangular, their corresponding sides are in proportion.

equiangular \triangle

36b) (CONVERSE) If the corresponding sides of two triangles are proportional, then their corresponding angles are equal.

sides of \triangle s prop.

37) The perpendicular drawn from the vertex of a right angle of a right-angled triangle to the hypotenuse, divides the triangle into two triangles that are similar to each other and to the original triangle.

perp. from rt. \angle to hypotenuse

Frequently used formulae

F1) Area rectangle = base \times height

F2) Area \triangle = $\frac{1}{2}$ base \times height

F3) Circumference of circle = $2\pi r$

F4) Area of circle = πr^2

F5) Area parallelogram = base \times height

F6) Area trapezium = $\frac{1}{2} \times$ (sum of parallel sides) \times height

F7) Area kite = $\frac{1}{2} \times$ diagonal \times diagonal

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Euklidiese meetkunde

Stellings en formules

Stellings

STELLING	AFKORTING
1a) As twee reguit lyne mekaar sny, is die som van enige paar aangrensende hoeke gelyk aan 180° .	gestrekte \angle
1b) (OMGEKEERDE) As die som van enige paar aangrensende hoeke gelyk is aan 180° , dan lê hul nie-gemeenskaplike sye op dieselfde reguit lyn.	aangr. \angle e suppl.
2) As twee reguit lyne mekaar sny, is die regoorstaande hoeke gelyk.	regoorst. \angle e
3a i) As 'n dwarslyn twee parallelle lyne sny, dan is die ooreenkomsstige hoeke gelyk.	\parallel lyne, ooreenk. \angle e
3a ii) As 'n dwarslyn twee parallelle lyne sny, dan is die verwisselende hoeke gelyk.	\parallel lyne, verw. \angle e
3a iii) As 'n dwarslyn twee parallelle lyne sny, dan is die som van die binnehoeke aan dieselfde kant van die dwarslyn, gelyk aan 180° .	\parallel lyne, ko-binne \angle e
3b i) (OMGEKEERDE) As twee lyne deur 'n dwarslyn gesny word, sodanig dat 'n paar ooreenkomsstige hoeke gelyk is, dan is die lyne parallel.	ooreenk. \angle e gelyk
3b ii) (OMGEKEERDE) As twee lyne deur 'n dwarslyn gesny word, sodanig dat 'n paar verwisselende hoeke gelyk is, dan is die lyne parallel.	verw. \angle e gelyk
3b iii) (OMGEKEERDE) As twee lyne deur 'n dwarslyn gesny word, sodanig dat 'n paar ko-binnehoeke supplementêr is, dan is die lyne parallel.	ko-binne \angle e suppl.
4) Lyne wat parallel is aan dieselfde lyn, is parallel aan mekaar.	\parallel aan dieselfde lyn
5) Die buitehoek van 'n driehoek is gelyk aan die som van die twee teenoorstaande binnehoeke.	buite \angle van \triangle
6) Die som van die binnehoeke van 'n driehoek is gelyk aan 180° .	binne \angle e \triangle
7i) As twee sye en 'n ingeslotte hoek van een driehoek respektiewelik gelyk is aan twee sye en 'n ingeslotte hoek van 'n ander driehoek, dan is die twee driehoekte kongruent aan mekaar.	s \angle s
7ii) As twee hoeke en 'n sy van een driehoek respektiewelik gelyk is aan twee hoeke en die ooreenstemmende sy van 'n ander driehoek, dan is die twee driehoekte kongruent aan mekaar.	$\angle\angle$ s



STELLING	AFKORTING
7iii) As drie sye van een driehoek respektiewelik gelyk is aan drie sye van 'n ander driehoek, dan is die driehoeke kongruent aan mekaar.	sss
7iv) As die skuinssy en een sy van 'n reghoekige driehoek respektiewelik gelyk is aan die skuinssy en een sy van 'n ander reghoekige driehoek, dan is die twee driehoeke kongruent aan mekaar.	90° ss
8a) Die basishoeke van 'n gelykbenige driehoek is gelyk.	gelyke \angle e teenoor gelyke sye
8b) (OMGEKEERDE) As twee hoeke van 'n driehoek gelyk is, is die sye teenoor die hoeke ook gelyk.	gelyke sye teenoor gelyke \angle e
9a) Die teenoorstaande sye van 'n parallelogram is gelyk.	teenoorst. sye \parallel^m
9b) (OMGEKEERDE) As albei pare teenoorstaande sye van 'n vierhoek gelyk is, dan is dit 'n parallelogram.	teenoorst. sye vierh. gelyk
10a) Die teenoorstaande hoeke van 'n parallelogram is gelyk.	teenoorst. \angle e \parallel^m
10b) (OMGEKEERDE) As albei pare teenoorstaande hoeke van 'n vierhoek gelyk is, is dit 'n parallelogram.	teenoorst. \angle e vierh. gelyk
11) Elke hoeklyn halveer die oppervlak van die parallelogram.	hoeklyn halveer \parallel^m
12) 'n Parallelogram en 'n reghoek op dieselfde basis en tussen dieselfde ewewydige lyne, is gelyk in oppervlak.	dieselde basis en hoogte
13) Indien 'n driehoek en 'n reghoek op dieselde basis en tussen dieselde ewewydige lyne lê, is die driehoek se oppervlakte gelyk aan die helfte van die reghoek se oppervlakte.	dieselde basis en hoogte
14a) Die hoeklyne van 'n parallelogram halveer mekaar.	hoeklyne \parallel^m
14b) (OMGEKEERDE) Indien die hoeklyne van 'n vierhoek mekaar halveer, is dit 'n parallelogram.	hoeklyne van vierh. halveer
15) Indien een paar teenoorstaande sye van 'n vierhoek gelyk en ewewydig is, is die vierhoek 'n parallelogram.	een paar teenoorst. sye gelyk en \parallel
16) Die hoeklyne van 'n reghoek is gelyk aan mekaar.	hoeklyne van reghoek
17) Die hoeklyne van 'n ruit halveer mekaar reghoekig en halveer die hoeke van die ruit.	hoeklyne van ruit

STELLING	AFKORTING
18) Die lynstuk wat die middelpunte van twee sye van 'n driehoek verbind, is ewewydig aan die derde sy en gelyk aan die helfte van die derde sy.	middelpunte van sye
19) Die lyn deur die middelpunt van een sy van 'n driehoek, ewewydig aan die tweede sy, halver die derde sy.	lyn deur middelpunt, 2e sy
20) As drie of meer ewewydige lyne gelyke lynstukke van 'n dwarslyn afsny, dan sal hul gelyke lynstukke van enige dwarslyn af-sny.	lyne, gelyke afsnitte
21a) In 'n reghoekige driehoek is die kwadraat van die skuinssy gelyk aan die som van die kwadrate van die ander twee sye.	Pyth.
21b) (OMGEKEERDE) As die kwadraat van een sy van 'n driehoek gelyk is aan die som van die kwadrate van die ander twee sye, dan is die hoek teenoor die eerste sy, 'n regte hoek.	omgekeerde Pyth.
22a) Die lynstuk wat die middelpunt van 'n sirkel en die middelpunt van 'n koord verbind, is loodreg op die koord.	lyn deur midpt. \odot en midpt. koord
22b) (OMGEKEERDE) Die loodlyn uit die middelpunt van 'n sirkel na enige koord, halver die koord.	loodlyn uit midpt. na koord
22c) (AFLEIDING) Die middelloodlyn van 'n koord gaan deur die middelpunt van die sirkel.	middelloodlyn van koord
23) Die middelpuntshoek in 'n sirkel is twee maal die omtrekshoek wat deur dieselfde boog by enige punt op die omtrek onderspan word.	mid.pts. $\angle = 2 \times \angle$ op \odot^{omtr}
24a) Die omtrekshoek in 'n halwe sirkel is 90° .	\angle in halwe sirkel
24b) (OMGEKEERDE) As 'n koord van 'n sirkel 'n regte hoek by 'n punt op die omtrek van die sirkel onderspan, is die koord 'n middellyn van die sirkel.	koord onderspan 90°
24c) (OMGEKEERDE) Die sirkel wat die skuinssy van 'n reghoekige driehoek omskryf, gaan deur die hoekpunt van die regte hoek.	\odot met skuinssy as middellyn
25a) Omtrekshoeke in dieselfde sirkelsegment is gelyk.	\angle_e in dieselfde segm.
25b) (OMGEKEERDE) As 'n lynstuk wat twee punte verbind, gelyke hoeke by twee ander punte aan dieselfde kant daarvan onderspan, is die vier punte konsiklies.	lynstuk onderspan gelyke \angle_e
25c) (AFLEIDING) Gelyke koorde onderspan gelyke omtrekshoeke in dieselfde sirkel.	gelyke koorde, gelyke \angle_e
25d) (AFLEIDING) In twee gelyke sirkels onderspan gelyke koorde gelyke omtrekshoeke.	gelyke koorde, gelyke radiusse, gelyke \angle_e
26a) Die teenoorstaande hoeke van 'n koordevierhoek is supplementêr.	teenoorst. \angle_e koordevierh.



STELLING	AFKORTING
26b) (OMGEKEERDE) As twee teenoorstaande hoeke van 'n vierhoek supplementêr is, is die vierhoek 'n koordevierhoek.	teenoorst. \angle e suppl.
27a) Die buitehoek van 'n koordevierhoek is gelyk aan die teenoorstaande binnehoek.	buite \angle koordevierh.
27b) (OMGEKEERDE) As 'n buitehoek van 'n vierhoek gelyk is aan die teenoorstaande binnehoek, is die vierhoek 'n koordevierhoek.	buite \angle = teenoorst. binne \angle
28a) 'n Raaklyn aan 'n sirkel is loodreg op die radius wat na die raakpunt getrek word.	raaklyn \perp radius
28b) (OMGEKEERDE) 'n Reguitlyn wat loodreg op die radius by 'n punt op die omtrek getrek word, is 'n raaklyn aan die sirkel.	lyn \perp radius
29) As twee raaklyne aan 'n sirkel vanuit dieselfde punt buite die sirkel getrek word, is die twee raaklyne ewe lank.	raaklyne vanuit dieselfde punt buite \odot
30a) Die hoek gevorm deur die raaklyn aan 'n sirkel en 'n koord wat vanuit dieselfde raakpunt getrek is, is gelyk aan die hoek in die teenoorstaande sirkelsegment.	\angle tussen raaklyn en koord
30b) (OMGEKEERDE) As 'n lyn vanuit die eindpunt van 'n koord getrek word sodat die hoek wat die lyn met die koord maak gelyk is aan 'n omtrekshoek in die teenoorstaande sirkelsegment, is die lyn 'n raaklyn aan die sirkel.	\angle tussen koord en lyn = \angle in teenoorst. segm.
GEMEENSKAPLIKE SNYPUNTE	
31) Die halveerlyne van die hoeke van 'n driehoek sny in een punt. (Die snypunt is die middelpunt van die ingeskreve sirkel van die driehoek.)	halveerlyne van \angle e van \triangle
32) Die middellooddlyne van die sye van 'n driehoek sny in een punt. (Die snypunt is die middelpunt van die omgeskrewe sirkel, ommiddelpunt, van die driehoek.)	middellooddlyne van sye van \triangle
33a) Die mediane (swaartelyne) van 'n driehoek sny in een punt. (Die snypunt is die swaartepunt van die driehoek.)	mediane van \triangle
33b) (AFLEIDING) Die swaartepunt is die derde-verdelingspunt van elke mediaan.	mediane van \triangle
34) Die hoogtelyne van 'n driehoek sny in een punt. (Die snypunt is die ortosnypunt van die driehoek.)	hoogtelyne van \triangle

In enige driehoek is die swaartepunt, ortosnypunt en om-middelpunt kolleïer. Die lyn deur dié drie punte word die Euler-Lyn genoem.

VERHOUDING EN EWEREDIGHEID

35a) 'n Reguit lyn parallel aan een sy van 'n driehoek verdeel die ander twee sye in eweredige dele.

lyn || een sy van \triangle

35b) (OMGEKEERDE) As 'n lyn twee sye van 'n driehoek in eweredige dele verdeel, is dié lyn parallel aan die derde sy.

lyn verdeel twee sye van \triangle eweredig

36a) As twee driehoeke gelykhoekig is, is hulle ooreenstemmende sye eweredig.

gelykhoekige \triangle e

36b) (OMGEKEERDE) As die ooreenstemmende sye van twee driehoeke eweredig is, is die driehoeke gelykhoekig.

sye van \triangle e eweredig

37) Die loodlyn vanuit die regtehoekpunt van 'n reghoekige driehoek op die skuinssy, verdeel die driehoek in twee driehoeke wat gelykvormig aan mekaar en gelykvormig aan die oorspronklike driehoek is.

loodlyn uit regte \angle op skuinssy

Algemene formules

F1) Oppv. reghoek = basis \times hoogte

F2) Oppv. \triangle = $\frac{1}{2}$ basis \times hoogte

F3) Omtrek van sirkel = $2\pi r$

F4) Oppv. van sirkel = πr^2

F5) Oppv. parallelogram = basis \times hoogte

F6) Oppv. trapesium = $\frac{1}{2} \times$ (som van ewewydige sye) \times hoogte

F7) Oppv. vlieër = $\frac{1}{2} \times$ hoeklyn \times hoeklyn

Bestel jou volledige gids:

MEETKUNDE = PRET

Graad 12

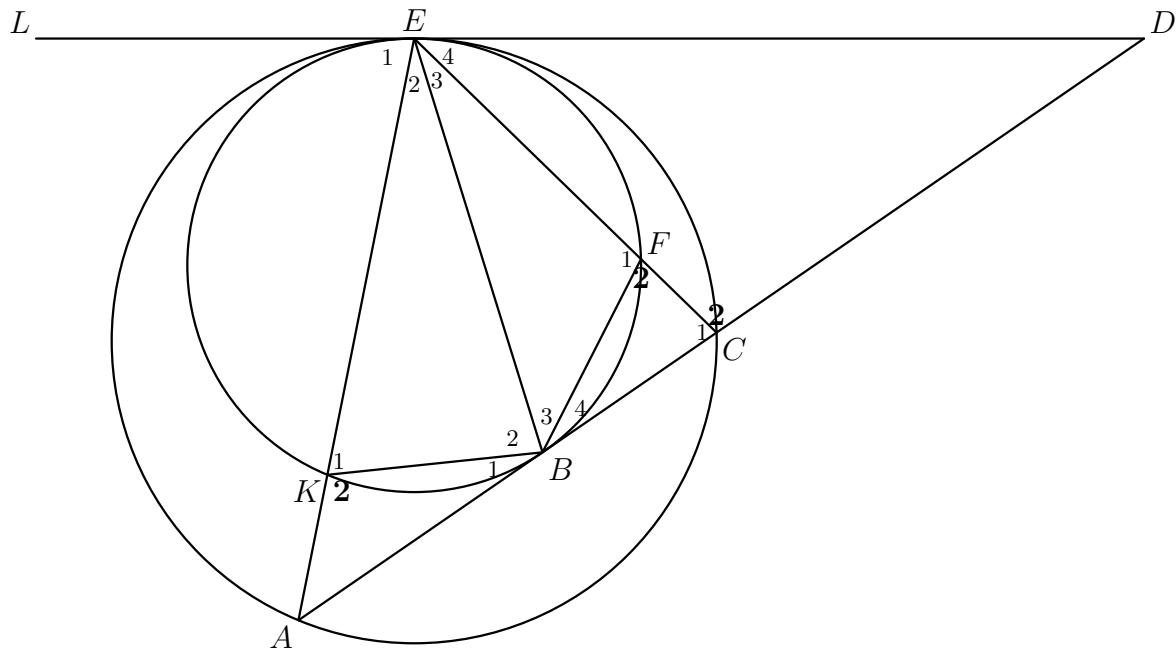
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Example



In the diagram, LED is the common tangent to the two circles that touch internally at E . AD is a tangent to the smaller circle at B , intersecting the bigger circle at A and C . EA , EB and EC are drawn. EC intersects the smaller circle in F . EA intersects the smaller circle in K . KB and BF are drawn.

Without drawing any additional lines, name

- Two pairs of equal angles, using only the theorem: An exterior angle of a cyclic quadrilateral is equal to the interior opposite angle
- Ten pairs of equal angles, using only the theorem: The angle between a tangent and a chord drawn from the point of contact is equal to an angle in the alternate segment of the circle.

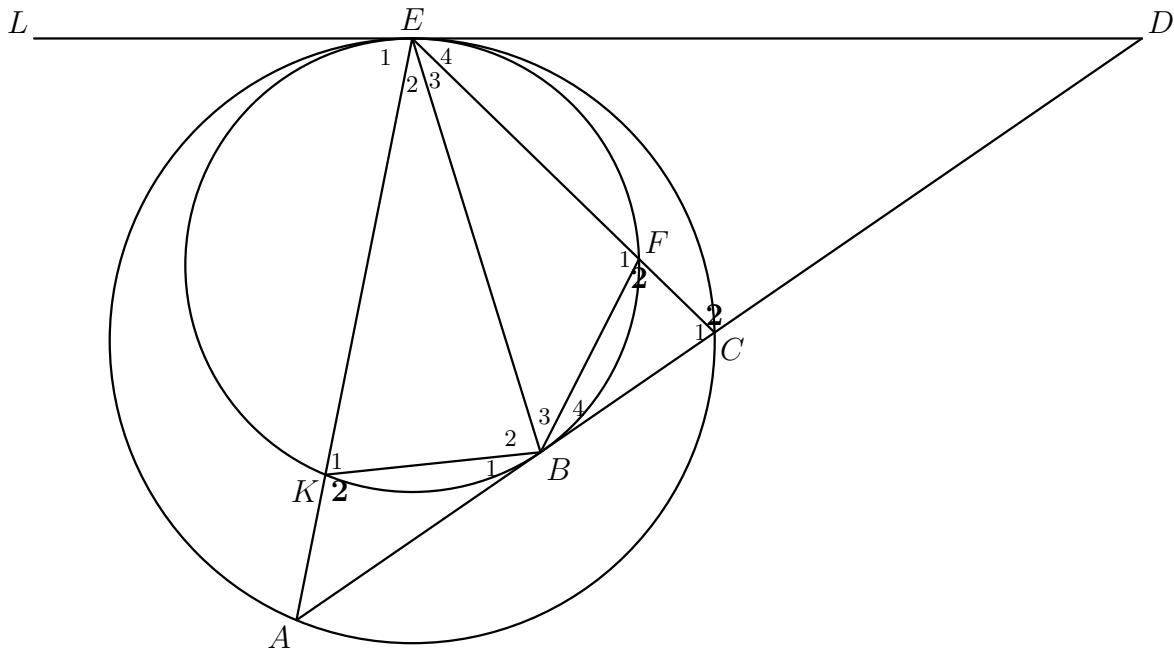
ANSWER a):

- $\hat{K}_2 = \hat{F}_1$
- $\hat{F}_2 = \hat{K}_1$

ANSWER b):

- | | |
|-------------------------------|--------------------------------|
| (i) $\hat{E}_4 = \hat{B}_3$ | (vi) $\hat{B}_4 = \hat{E}_3$ |
| (ii) $\hat{E}_4 = \hat{A}$ | (vii) $B\hat{E}L = \hat{F}_1$ |
| (iii) $B\hat{E}D = \hat{K}_1$ | (viii) $E\hat{B}C = \hat{K}_1$ |
| (iv) $\hat{E}_1 = \hat{B}_2$ | (ix) $\hat{B}_1 = \hat{E}_2$ |
| (v) $\hat{E}_1 = \hat{C}_1$ | (x) $A\hat{B}E = \hat{F}_1$ |

Voorbeeld



In die diagram is LED die gemeenskaplike raaklyn aan twee sirkels wat mekaar intern raak by E . AD is 'n raaklyn aan die kleiner sirkel by B en dit sny die groter sirkel by A en C . EA , EB en EC word ingeteken. EC sny die kleiner sirkel by F . EA sny die kleiner sirkel by K . KB en BF word ingeteken.

Sonder om addisionele lyne te trek, noem

- Twee pare gelyke hoeke deur gebruik te maak van die stelling: Die buitehoek van 'n koordevierhoek is gelyk aan die teenoorstaande binnehoek
- Tien pare gelyke hoeke deur gebruik te maak van die stelling: Die hoek gevorm deur die raaklyn aan 'n sirkel en 'n koord wat vanuit dieselfde raakpunt getrek is, is gelyk aan die hoek in die teenoorstaande sirkelsegment.

ANTWOORD a):

- $\hat{K}_2 = \hat{F}_1$
- $\hat{F}_2 = \hat{K}_1$

ANTWOORD b):

- | | |
|-------------------------------|--------------------------------|
| (i) $\hat{E}_4 = \hat{B}_3$ | (vi) $\hat{B}_4 = \hat{E}_3$ |
| (ii) $\hat{E}_4 = \hat{A}$ | (vii) $B\hat{E}L = \hat{F}_1$ |
| (iii) $B\hat{E}D = \hat{K}_1$ | (viii) $E\hat{B}C = \hat{K}_1$ |
| (iv) $\hat{E}_1 = \hat{B}_2$ | (ix) $\hat{B}_1 = \hat{E}_2$ |
| (v) $\hat{E}_1 = \hat{C}_1$ | (x) $A\hat{B}E = \hat{F}_1$ |

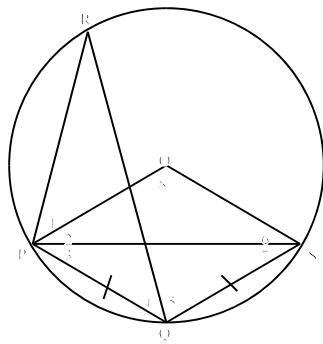


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Worksheet / Werkkaart 19

1. In the figure, O is the centre of the circle and P, Q, R and S are points on the circumference such that $PQ = QS$. If $\hat{O}_8 = x$, express the following angles in terms of x .
(Give reasons for your answers.)

In die figuur is O die middelpunt van die sirkel en P, Q, R en S is punte op die omtrek sodat $PQ = QS$. As $\hat{O}_8 = x$, druk die volgende hoeke uit in terme van x .
(Gee redes vir u antwoorde.)



1.1 \hat{P}_2 (4) 1.2 \hat{PQS} (2) 1.3 \hat{R} (7)

2. The diagonals PR and QS of square PSRQ, with sides 6 units long, intersect at K. T is the midpoint of SR and PT cuts QS at G.

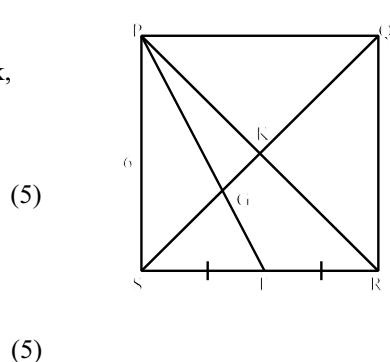
Die hoeklyne PR en QS van vierkant PSRQ, met sye 6 eenhede lank, ontmoet in K. T is die middelpunt van SR en PT sny QS in G.

- 2.1 Prove that G is the centroid of $\triangle PSR$.

Bewys dat G die swaartepunt van $\triangle PSR$ is.

- 2.2 Calculate the length of PG, leaving your answer in simplest surd form.

Bereken die lengte van PG en laat u antwoord in eenvoudigste wortelvorm.



(5)

3. In the figure, ABCF is a cyclic quadrilateral with $AB = BF$ and AF and BC, both produced, meet at E. The tangent to the circle at F meets BE at D.

In die figuur is ABCF 'n koordevierhoek met $AB = BF$ en AF en BC, beide verleng, ontmoet in E. Die raaklyn aan die sirkel by F ontmoet BE in D.

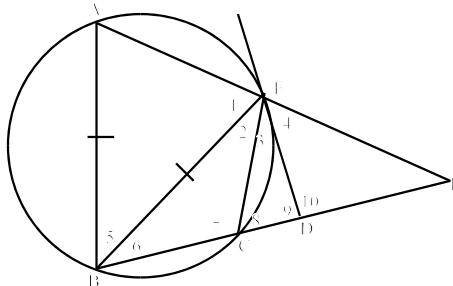
- 3.1 Prove that / Bewys dat:

3.1.1 $\hat{F}_1 = \hat{C}_8$ (4)

3.1.2 $\hat{F}_4 = \hat{B}_5$ (4)

- 3.1.3 FB is a tangent at F to the circle passing through the points F, C and E.
FB 'n raaklyn by F aan die sirkel deur die punte F, C en E is.

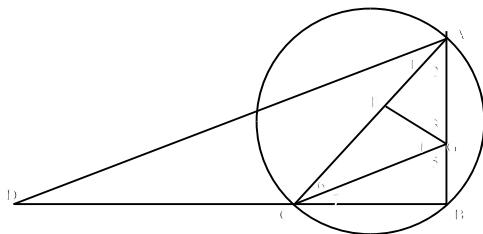
(5)



- 3.2 If / As $\hat{E} = 30^\circ$ and / en $\hat{F}_4 = 40^\circ$, show that / bewys dat $\hat{F}_3 = \hat{F}_4$ (4)

4. In the figure, AC is a diameter of circle ABC. The bisector CG of $A\hat{C}B$ meets AB at G and GF is perpendicular to AC. The straight line AD is parallel to GC and meets BC produced at D.

In die figuur is AC 'n middellyn van sirkel ABC. CG, die halveerlyn van $A\hat{C}B$, ontmoet AB in G en GF is loodreg op AC. Die reguitlyn AD is ewewydig aan GC en ontmoet BC verleng in D.



4.1 Prove that / Bewys dat:

4.1.1 $\triangle CAD$ is isosceles / gelykbenig is. (4)

$$4.1.2 \frac{BC}{AC} = \frac{BG}{GA} \quad (3)$$

4.1.3 $\triangle AFG \parallel\!/\!\!/ \triangle ABC$ (4)4.2 If $BC = 12$ units and the radius of the circle is 10 units, calculate:4.2 As $BC = 12$ eenhede en die radius van die sirkel is 10 eenhede is, bereken:

4.2.1 AB (4)

4.2.2 AG (5)

4.2.3 AF (3)

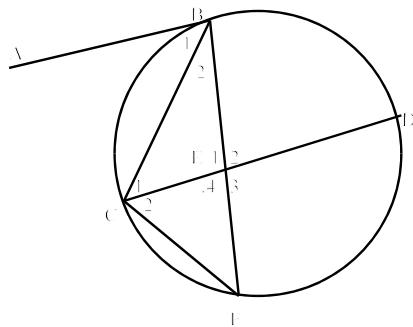
5. In the figure, AB is a tangent to the circle at B . Chords CD and BF intersect at E so that BC is a tangent to the circle through the points E, C and F . C and F are joined.

In die figuur is AB 'n raaklyn aan die sirkel by B . Koorde CD en BF sny mekaar in E sodat BC 'n raaklyn aan die sirkel deur die punte E, C en F is. C en F is verbind.

Prove that / Bewys dat:

5.1 $AB \parallel CD$ (7)

5.2 $BF = \frac{BC \cdot FC}{CE}$ (6)



6. In the figure, the two circles touch at P . QP is a common tangent to the circles. QBA , QCD and PCB are straight lines. PA , PD and BD are line segments.

In die figuur raak die twee sirkels mekaar in P . QP is 'n gemeenskaplike raaklyn aan die sirkels. QBA , QCD en PCB is reguitlyne. PA , PD en BD is lynstukke.

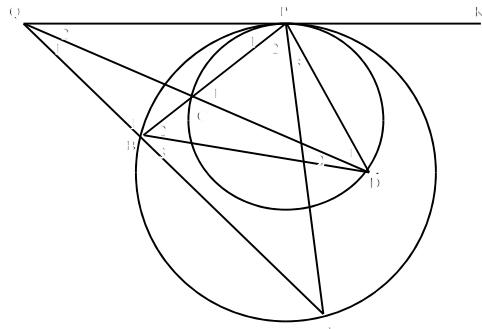
Prove that the following are cyclic quadrilaterals:

Bewys dat die volgende koordevierhoeke is:

6.1 $PQAD$ (3)

6.2 $CBAD$ (4)

6.3 $PBAD$ (2)

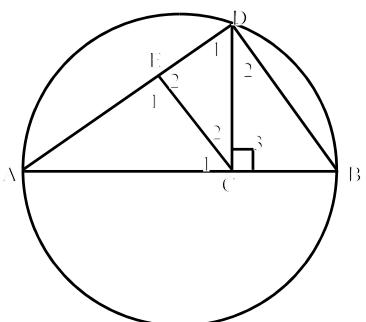


7. In the figure, AB is a diameter and AD and DB are chords of the circle. C is a point on AB such that $DC \perp AB$. E is a point on AD such that:

In die figuur is AB 'n middellyn en AD en DB koorde van die sirkel. C is 'n punt op AB sodanig dat $DC \perp AB$. E is 'n punt op AD sodanig dat:

$$\frac{AE}{ED} = \frac{AC^2}{CD^2}$$

Prove that / Bewys dat:



7.1 $\triangle DAB \parallel\!/\!\!/ \triangle CDB \parallel\!/\!\!/ \triangle CAD$ (3)

7.2 $EC \parallel DB$ (6)



Solutions / Oplossings

PAPER 1

Worksheet / Werkkaart 1

1. a) $2x^2 - 3x$
- b) $-2x^2 - x$
- c) $-2x^2 + 4x^3$
- d) $-2x^2 + 4x^3$
- e) $-2x^2 - x$
- f) $-2x^2 + 4x^3$
- g) $8x^3 - 12x^2y + 6xy^2 - y^3$
- h) $x^3 + 8$
- i) $8x^3 - 27$
2. a) $(x + 4)(x - 3)$
- b) $(x - 4)(x + 3)$
- c) cannot factorise
- d) cannot factorise
- e) $(x - 4)(x - 3)$
- f) $(x + 4)(x + 3)$
- g) $(x - 6)(x + 1)$
- h) $7(x - 2)(x^2 + 2x + 4)$
- i) $2(3 - 2x + 2y)(3 + 2x - 2y)$
- j) $(2x - y)(2x - y - 4)$
- k) $(x - 2y)(2x + 4y - 1)$
- l) $(x - 3)(x - 4)(x + 4)$
- m) $(x - 2)(x - 5)(x + 3)$
- n) $(x + 1)(x - 2)^2$
3. a) $\frac{-3x^2}{5x(x - 5)(x + 5)}$
- b) $\frac{a^2 - ab + b^2}{a - b}$

c) $\frac{-x - 1}{x(x - 2)}$
d) $\frac{x^3 + 2x^2 + 1}{(x - 1)(x + 1)}$

Worksheet / Werkkaart 2

1. a) False
- b) False
- c) False
2. a) $\frac{9}{4}$
- b) $-\frac{7}{16}$
- c) $27 \frac{1}{4}$
- d) $\frac{85}{17}$
- e) $\frac{1}{16}$
- f) 64
3. a) $\sqrt{2}$
- b) $12\sqrt{2}$
- c) 4
- d) $-\frac{1}{2}$
- e) 4
- f) $3\sqrt{2}$
- g) 3
4. a) $\frac{\sqrt{2}}{2}$
- b) $\frac{5\sqrt{3}}{-6}$

Worksheet / Werkkaart 3

1. a) $1\frac{1}{2}$ or -2
 b) $-1\frac{1}{2}$ or 4
 c) ± 5
 d) 6 or -4
 e) ± 4
 f) $4\frac{1}{2}$ or $\frac{2}{3}$
 g) -8
 h) 3 or $10\frac{2}{3}$
 i) no solution
2. a) -3 or -4
 b) no real solution
 c) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 d) $x = \frac{1 \pm \sqrt{4m^2 + 8m + 5}}{2(m+1)}$
3. a) $x = \pm \sqrt{\frac{3}{2}}$
 b) $1,24$ or $-3,23$
4. a) $(x-3)(x^2 + 4)$
 b) $(x-2)(2x-1)(x+3)$
 c) $(x-2)(x+7)(x+1)$
 d) $(x+3)(x-\sqrt{3})(x+\sqrt{3})$
5. a) $x = \frac{3}{4}$ or $x = 1$ or $x = -1$
 b) $x = 1$ or $x = -2$
 c) $x = 1$ or $x = 6$ or $x = -2$
6. a) no solution
 b) $-\frac{3}{2}$
7. $-1 \pm 2i$

d) 1,58

e) 1,75

f) 0,63

g) 16

h) 2

i) 5

2. $n = \log_{(1+i)}\left(\frac{A}{P}\right)$

3. $n = \log_{(1+i)}\left(\frac{F_i}{x} + 1\right)$

4. (1; 0) or (5; 8)

5. a) (4; -2)

b) (-1; 1) or ($\frac{11}{7}; \frac{13}{7}$)c) ($\frac{2}{3}; \frac{3}{2}$) or ($\frac{1}{2}; 1$)d) (0,5; 1) or ($\frac{2}{3}; \frac{2}{3}$)6. a) $x < \frac{1}{2}$ b) $x < -1$ or $x > 2$ c) $\frac{1}{2} < x < \frac{5}{2}$ d) $x \leq -2$ or / of $x \geq 3$ e) $-3 < x < 4$ f) $-2 \leq x \leq 4$ g) $-4 < x < 4$ h) $x \leq -3$ or / of $x \geq 3$ **Worksheet / Werkkaart 4**

1. a) 5
 b) 2,32
 c) 2,86

Worksheet / Werkkaart 5

1. 2; 3; 5; 7; 11; 13; 17; ...

Product 1st; 2nd; 3rd; ... $= 2; 6; 30; 210; 2310; \dots$

∴ Units digit is 0

2. a) $90(0,7)^5 = 15,126 \text{ m}$

b) 11 bounces

c) 510 m

3. a) 29

b) 1; 5; 11; 19; 29; ...

4; 6; 8; 10; ...

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2; 2; 2; ...

$$\therefore T_n = \left(\frac{T_1}{2} \right) n^2 + qn + r$$

Substitute $T_1 = 1$ & $T_2 = 5$:

$$\therefore T_n = 0,5n^2 + 2,5n - 2$$

c) $T_{50} = 1373$

4. a) 6×3 pool: $2(6+2) + 2(3) = 22$

8×5 pool: $2(8+2) + 2(5) = 30$

b) $m \times n$ pool:

$$2(m+2) + 2(n) = 2(m+n) + 4$$

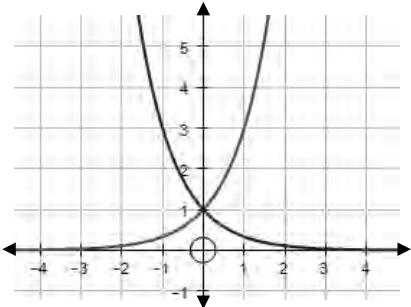
c) $m \times n$ pool:

$$2(m+m) + 4 = 4m + 4$$

d) 49×49 pool

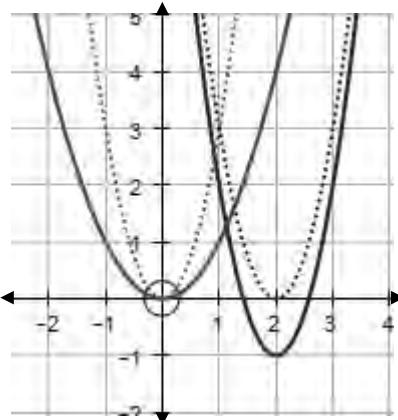
Worksheet / Werkkaart 7

1. a) 2
b) 360°
c) $[-2; 2]$
d) 2
2. a) -2
b) $x = 90^\circ + k \cdot 180^\circ$
c) 180°
d) R
3. a) 11
b) no x -intercepts
c) 2
d) R
e) $[2; \infty)$
f) $x = 3$
4. a)



- b) $x = 0$

5. a) & c)



a)

b)

c)

d)

- b) $3 \times$ vertical stretch

Translation 2 units to the right

Translation 1 unit downwards

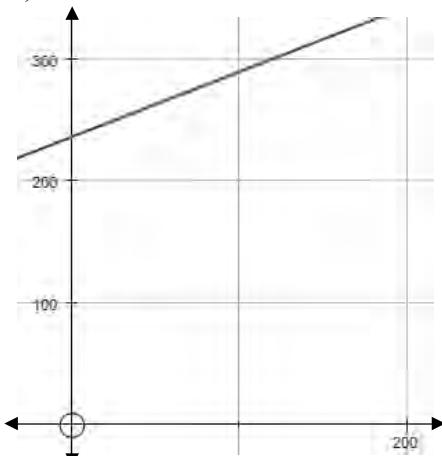
d) R

e) $[-1; \infty)$ **Worksheet / Werkkaart 8**

1. a) $C(x) = \frac{8}{15}x + 236$

b) $C(2000) = \text{R}1302,67$

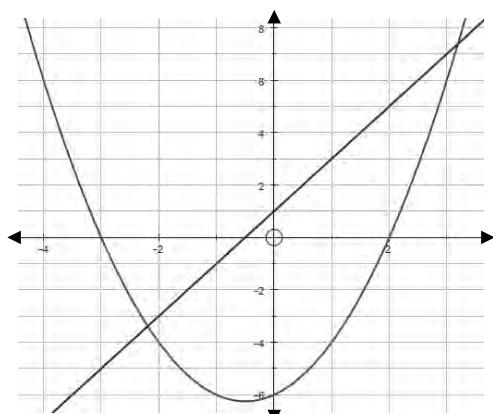
c)



d) The rate that the cost change with respect to the units used.

e) Basic cost

2.



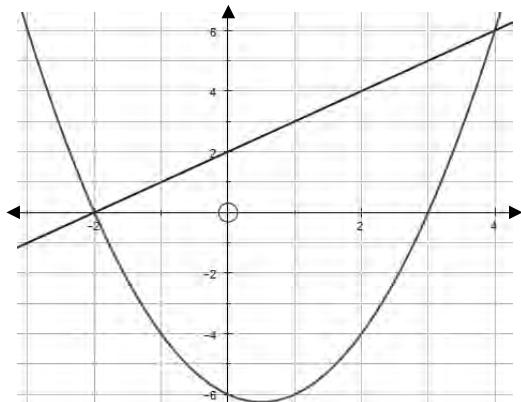
a) $x = -3 \text{ or } x = 2$

b) $x \leq -3 \text{ or } -\frac{1}{2} < x \leq 2$

c) $-3 \leq x \leq -\frac{1}{2} \text{ or } x \geq 2$

d) $x = -\frac{1}{2}$

3. a)



b) $x^2 - 2x - 8 = 0$

$\Leftrightarrow x^2 - x - 6 = x + 2$

$\therefore (-2; 0) \text{ & } (4; 6)$

Worksheet / Werkkaart 9

1. a) A(0; 9)

B(1; 0)

C(9; 0)

b) OD = 5

DE = 16

AF = 10

HI = 20

2. $f(x) = 2 \sin(\frac{1}{2}x) + 2$

$g(x) = 4\tan x$

Worksheet / Werkkaart 10

1. $m = 1$

2. a) $m = 1$

b)
$$\frac{-2}{(a+h-1)(a-1)}$$

3. $-8x$

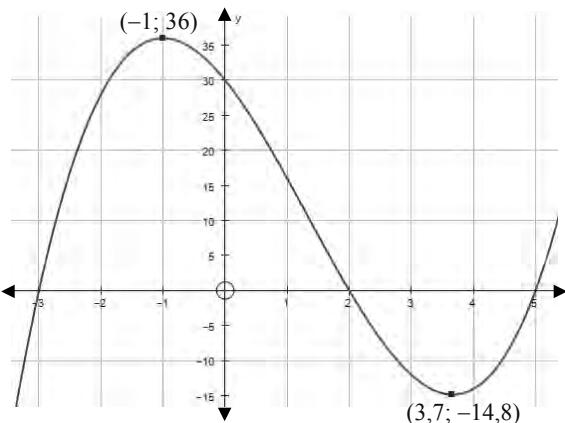
4. a) $f'(x) = 3x^2 - 2x + 2$

b) $f'(x) = 2 + \frac{8}{x^3}$

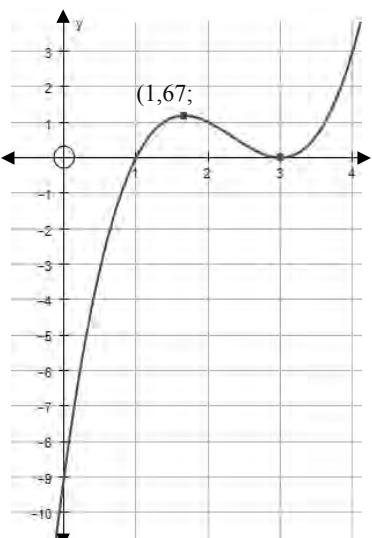
c) $f'(x) = \frac{-12}{x^4} + \frac{1}{2\sqrt{x}}$

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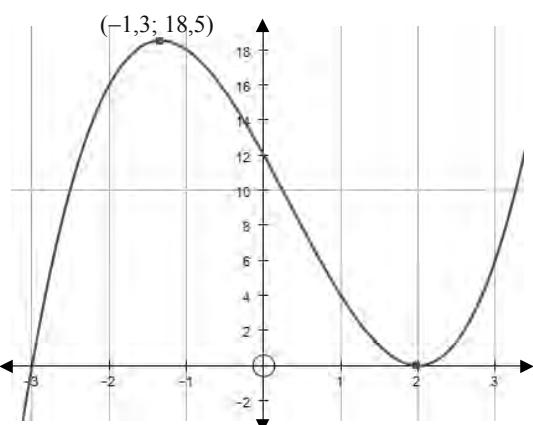
- d) $f'(x) = 9x^2 + 4x - 4$
e) $f'(x) = \frac{1}{2\sqrt{x}} + 2x + 6$
5. a) $y = -2x - 3$ & $y = 2x - 7$
b) $(1; -5)$
6. a)



b)



c)



7. Volume = $\pi r^2 h \therefore h = \frac{350}{\pi r^2}$

$$A(r) = 2(\pi r^2) + 2\pi r h$$

$$A'(r) = 4\pi r - \frac{700}{r}$$

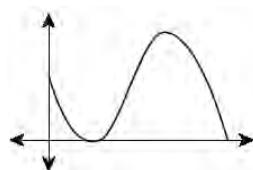
$$\therefore r = 3,81 \text{ cm} \text{ & } h = 7,67 \text{ cm}$$

Worksheet / Werkkaart 11

1. a) $B(0) = 1200$
b) $B'(t) = -40$
c) Decrease with 40 million per hour.

- d) Shrinking
e) $t = 6$
f) 6 hours

2. a) $H(0) = 8$
b) $t = 2^\circ\text{C}$ or/of $t = 20^\circ\text{C}$
c) $t = 14^\circ\text{C}$
d)



3. a) $SR = (72 - 2x) \text{ m}$
b) $A = 72x - 2x^2$
c) $x = 18 \text{ m}$
d) $A = 648 \text{ m}^2$

PAPER 2

Worksheet / Werkkaart 12

1. $\frac{1}{2}n^2 - \frac{3}{2}n$

2.

5	100	n
540°	17640°	180°(n - 2)
108°	176,4°	$\frac{180^\circ(n-2)}{n}$

3. 150°

4 a) $441,1 \text{ cm}^2$

b) $412,6 \text{ cm}^2$

c) $414,7 \text{ cm}^2$

d) $H = \sqrt{91}$

$\pi r^2 + \pi r H$

$= \pi(3)^2 + \pi(3)\sqrt{91}$

$= 118,2 \text{ cm}^2$

e) $(8)(6) + 2\left(\frac{1}{2}(6)(\sqrt{116})\right) + 2\left(\frac{1}{2}(8)(\sqrt{109})\right)$

$= 196,1 \text{ cm}^2$

Worksheet / Werkkaart 13

1. a) $(-6; 4)$

b) $t = 6$

2. a) $x^2 + y^2 = 40$

b) $q = \pm 2$

c) $\theta = 26,6^\circ$

3. a) $(1; -2) \& r = \sqrt{20}$

b) $m_{\text{radius}} = -2$

$m_{\text{tangent}} = \frac{1}{2}$

$y + 6 = \frac{1}{2}(x - 3)$

$y = \frac{1}{2}x - \frac{15}{2}$

4. $m_{AC} = -1 \& m_{BC} = \frac{4}{9}$

Angle $= 45^\circ + 23,9^\circ = 68,9^\circ$

5. a) $x^2 - 8x + y^2 - 4y - 12 = 0$

b) $(0; 6); (0; -2)$

6. $r = \frac{2}{3}$

Worksheet / Werkkaart 14

1. $(x - 5)^2 + (y + \frac{1}{2})^2 = 25\frac{1}{4}$

$M(5; -\frac{1}{2}) \& r = 5,025$

2. a) $m_{AB} = \frac{2}{3}$

b) $e = -3$

c) $3x + 2y - 7 = 0$

3. a) $\sqrt{26} + \sqrt{26} + \sqrt{52} = 17,4$

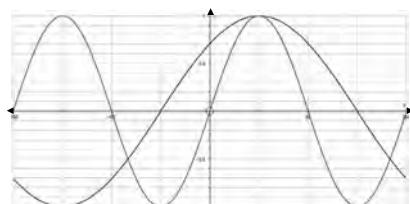
b) $11,3^\circ$

c) $(-3,5; -0,5)$

d) $y = 0,2x + 0,2$

Worksheet / Werkkaart 15

1. a)

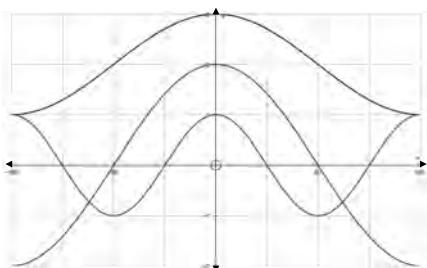


b) $f(x): 180^\circ \& g(x): 360^\circ$

2. a)

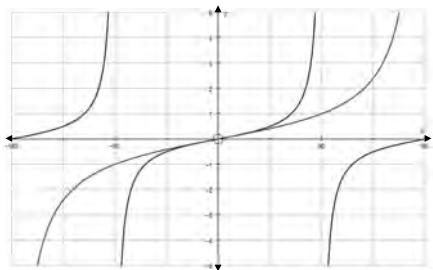


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Worksheet / Werkkaart 16

- b) $x = -135^\circ; -45^\circ; 45^\circ; 135^\circ$
c) $2 - \cos x = 2 + \cos x - 2 \cos x$
 $= (2 + \cos x) - (2 \cos x)$
 $x = -180^\circ; 180^\circ$

3. a)



- b) 180°
c) $x = 0^\circ$
d) $-180^\circ < x < -90^\circ$ or $0^\circ < x < 90^\circ$

4. a = 2 & k = -1

b = 1 & m = -1

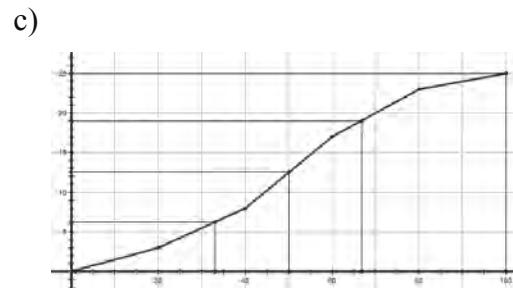
1. a) 1
b) $-2\sqrt{3}$
c) 3
d) -2
e) -3
f) 2
2. a) t
b) t
c) $\frac{-t}{\sqrt{1-t^2}}$

3. a) 150°
b) $x \in \mathbb{R}$
c) $x = 46,3^\circ$
4. a) $12,3^\circ; 107,7^\circ$
b) $212,4^\circ; 7,6^\circ$
5. a) 555°
b) $\frac{-\sqrt{3}}{3}$
6. a) $x = k \cdot 180^\circ$
 $x = 30^\circ + k \cdot 360^\circ$
 $x = 150^\circ + k \cdot 360^\circ$
 $k \in \mathbb{Z}$
b) $\theta = 60^\circ + k \cdot 360^\circ$
 $\theta = 300^\circ + k \cdot 360^\circ$
 $k \in \mathbb{Z}$
7. Prove LHS = RHS
8. $\frac{181}{65}$

Worksheet / Werkkaart 17

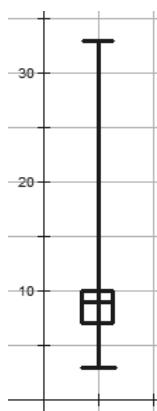
1. a) $a = 5$ or $a = 3$
 b) Area = $10\sqrt{3}$
2. a) $\Delta PQT \equiv \Delta PRT$ (S \angle S)
 $\therefore PQ = PR$
 b) $PQ = \frac{y}{\cos \alpha}$; Use sine rule
 c) a) 12°
 b) $\sin 19^\circ = \frac{30}{TB}$
 $TB = 9,77$ m
 c) $\frac{BC}{\sin 7^\circ} = \frac{9,77}{\sin 12^\circ}$
 $BC = 5,73$ m

3. a) i) 33
 ii) 67
 iii) 34
 iv) 17
 b)
- | | | |
|--------|---|----|
| 1-20 | 3 | 3 |
| 21-40 | 5 | 8 |
| 41-60 | 9 | 17 |
| 61-80 | 6 | 23 |
| 81-100 | 2 | 25 |

**Worksheet / Werkkaart 18**

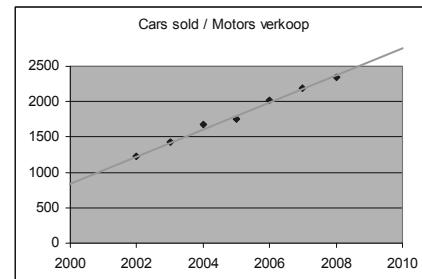
1. a) 10,82
 b) 9
 c) 9
 d) 30
 e) 7
 f) 10
 g) 317
 h) 8,15
 i) 66,4

2.



d) 50

4. a)



b) Linear

c) 3000

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Worksheet / Werkkaart 19

1.1 $\hat{P}_2 = \hat{S}_6$ OP = OS ; radii
 $= \frac{180^\circ - x}{2}$ $\angle s$ of \triangle / $\angle e$ van \triangle

1.2 Reflex/ Inspringende \angle POS = $360^\circ - x$
 $P\hat{Q}S = \frac{360^\circ - x}{2}$ central-inscribed \angle
mid.punt-omtreks \angle

1.3 $\hat{R} = \hat{S}_7$ subtended by PQ
onderspan deur PQ
 $= \hat{P}_3$ PQ = QS
 $= \frac{180^\circ - P\hat{Q}S}{2}$ $\angle s$ of \triangle / $\angle e$ van \triangle
 $= \frac{x}{4}$

2.1 PK = KR diagonals of square
hoeklyne van vierkant
SK median of / swaartelyn van \triangle PSR
Also / Ook ST = TR given / gegee
 \therefore PT median of / swaartelyn van \triangle PSR
 \therefore G centroid of / swaartepunt van \triangle PSR

2.2 PT = $3\sqrt{5}$ Pythagoras
PG = $\frac{2}{3}$ PT G centroid / swaartepunt
 $= 2\sqrt{5}$

3.1.1 $\hat{F}_1 = \hat{A}$ AB = BF
 $= \hat{C}_8$ ext. \angle cyclic quad

3.1.2 $\hat{F}_3 = \hat{B}_6$ tangent, chord
raaklyn, koord

$\hat{F}_3 + \hat{F}_4 = \hat{B}_6 + \hat{B}_5$ ext. \angle cyclic quad
buit e \angle kvh

$\hat{F}_4 = \hat{B}_5$

3.1.3 $\hat{F}_1 + \hat{F}_2 = \hat{C}_8 + \hat{E}$ ext. \angle / buite \angle \triangle FCE
But / maar:

$\hat{F}_1 = \hat{C}_8$ from / van 3.1

$\therefore \hat{F}_2 = \hat{E}$

\therefore BF a tangent / 'n raaklyn

3.2 $\hat{F}_4 = \hat{B}_5 = 40^\circ$ In \triangle ABF:
 $\hat{A} = \hat{F}_1$
 $= 70^\circ$ $\angle s$ of \triangle / $\angle e$ van \triangle

$\hat{C}_8 = \hat{A} = 70^\circ$ ext. \angle cyclic quad
buite \angle kvh

$\hat{D}_9 = \hat{F}_4 + \hat{E} = 70^\circ$ ext. \angle / buite \angle \triangle DEF

In \triangle CDF:
 $\hat{F}_3 = 180^\circ - (\hat{C}_8 + \hat{D}_9)$ $\angle s$ of \triangle / $\angle e$ van \triangle
 $= 40^\circ = \hat{F}_4$

4.1.1 $\hat{A}_1 = \hat{C}_6$ AD // CG ; alt/verw \angle
 $\hat{D} = \hat{C}_7$ AD // CG ; corr/ooreenk \angle
But/maar:
 $\hat{C}_6 = \hat{C}_7$ CG bisects \angle ACB
CG halveer \angle ACB
 $\therefore \hat{A}_1 = \hat{D}$
 $\therefore \triangle$ CAD isosceles / gelykbenig

4.1.2 In ABD: $\frac{BC}{CD} = \frac{BG}{GA}$ AD // CG
But/maar: CD = AC \triangle CAD isosceles / gelykbenig
 $\therefore \frac{BC}{AC} = \frac{BG}{GA}$

4.1.3 In \triangle AFG and/en \triangle ABC:
(i) $\hat{A} = \hat{A}$ Commom/Gemeensk.
(ii) $A\hat{F}G = 90^\circ$ GF \perp AC
 $= \hat{B}$ \angle in semi \odot
(iii) $\hat{G}_3 = \hat{C}_6 + \hat{C}_7$ $\angle s$ of \triangle / $\angle e$ van \triangle
 $\therefore \triangle$ AFG // \triangle ABC $\angle \angle \angle$

4.2.1 AC = 2.radius = 20 units/eenhede

In \triangle ABC:

$AB = \sqrt{20^2 - 12^2} = 16$ units/eenhede

4.2.2 $\frac{AG}{AB} = \frac{DC}{DB}$ AD // CG
 $\frac{AG}{16} = \frac{20}{32}$ DC = AC ; DB = DC+BC
 $AG = 10$ units / eenhede

4.2.3 From/Van: $\triangle AFG \sim \triangle ABC$

$$\frac{AF}{AB} = \frac{AG}{AC}$$

$$\frac{AF}{16} = \frac{10}{20}$$

$AF = 8$ units / eenhede

- 5.1 $\hat{C}_1 = \hat{F}$ tangent/raaklyn BC
chord/koord EC
 $= \hat{B}_1$ tangent/raaklyn AB
chord/koord BC
 $\therefore AB \parallel CD$ alt. \angle s = / verw. \angle e =

5.2 In $\triangle BFC$ and/en $\triangle BCE$:

(i) $\hat{B}_2 = \hat{B}_2$ Common/Gemeensk.

(ii) $\hat{F} = \hat{C}_1$ 5.1

(iii) $\hat{C}_1 + \hat{C}_2 = \hat{E}_1$ \angle s of \triangle / \angle e van \triangle

$\therefore \triangle BFC \sim \triangle BCE$ $\angle\angle\angle$

$$\therefore \frac{BF}{BC} = \frac{FC}{CE}$$

$$\therefore BF = \frac{BC \cdot FC}{CE}$$

- 6.1 $\hat{P}_1 = \hat{D}_1$ tangent/raaklyn QP
chord / koord PC

But/maar

$\hat{P}_1 = Q\hat{A}P$ tangent/raaklyn QP
chord / koord PB

\therefore Line QP subtends equal angles

Lynstuk QP onderspan gelyke hoeke

\therefore PQAD is a cyclic quad / is 'n koordev.

- 6.2 $Q\hat{A}D = K\hat{P}D$ ext. \angle cyclic quad PQAD
buite \angle kvh PQAD

$$= \hat{C}_1 \text{ tangent / raaklyn PK}$$

chord / koord PD

Ext. \angle = opp. interior \angle

Buite \angle = teenoorst. binne \angle

\therefore CBAD is a cyclic quad / is 'n koordev

6.3 For PBAD to be a cyclic quad:

Vir PBAD om 'n koordevierh. te wees, moet:

$$B\hat{A}P = \hat{D}_1 + \hat{D}_2$$

But/Maar $B\hat{A}P = \hat{D}_1$

\therefore PBAD is not a cyclic quad / is nie 'n koordev nie.

- 7.1 $A\hat{D}B = 90^\circ$ \angle in semi \odot

$$= \hat{C}_3 \text{ given / gegee}$$

$\therefore \triangle DBA \sim \triangle CBD \sim \triangle CDA$

two triangles similar to each other and similar to the original triangle.

twee driehoeke wat gelykvormig is aan mekaar en aan die oorspronklike driehoek.

7.2 $\triangle CDB \sim \triangle CAD$ 7.1

$$\frac{CD}{CA} = \frac{CB}{CD}$$

$$CD^2 = CA \cdot CB$$

$$\text{but / maar: } \frac{AE}{ED} = \frac{AC^2}{CD^2} \quad \text{given / gegee}$$

$$= \frac{AC^2}{CA \cdot CB}$$

$$= \frac{AC}{CB}$$

\therefore EC divides AD and AB proportionally

EC verdeel AD en AB in dieselfde verhouding

$\therefore EC \parallel DB$



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THE DIFFERENCE STARTS WITH OUR YOUTH.

#YOUNGDIFFERENCETAKER



The Nedbank External Bursary Programme is open to full-time Undergraduate and Honours students who are studying towards qualifications in the financial sector with a strong focus on STEM (Science, Technology, Engineering and Mathematics) and Green Economy skills.

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CRITERIA FOR SELECTION:

To be eligible for the Nedbank bursary, you must:

- be a South African citizen; and
- have a proven financial need, as determined by your household income.

Must have:

- a minimum academic average of 65% in the year that you are applying for the bursary;
- a minimum academic average of 65% for the required subjects aligned with the qualification you plan to study towards at university;
- not completed your grade 12 or senior certificate with mathematics literacy; and
- successfully obtained admission approval to a South African public university or university of technology for the undergraduate qualification you plan to study towards.

The application process:

We screen bursary applications against our external bursary selection criteria and award bursaries through the Nedbank Educational Trust, at the sole discretion of the trust.

The screening process:

- We will only consider bursary applications submitted online via the link we have provided.
- Incomplete bursary applications will be rejected.
- Bursary applications will be screened and shortlisted according to the qualifying criteria.

If you are shortlisted, we will ask you to:

- submit the relevant documents for financial review;
- participate in psychometric and other assessments; and
- attend an interview.

YOUR FUTURE STARTS NOW!

All candidates will receive feedback on the status of their application at the completion of the recruitment and selection process.

Contact information:

Email: Unlockyourambition@nedbank.co.za

**Closing date for applications is
31 August 2024.**

Bursaries are granted at Nedbank's sole discretion.

For the complete application process and a list of required documents, please visit our website at www.nedbank.co.za/bursary